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Abstract

This paper attempts to analytically determine the impact a tax shield (marginal tax rate) has on the value of a levered firm assuming that gains and losses are taxed differently. Previous research has done this by employing empirical methods and simulation studies. We are able to present closed-form solutions for two popular financing policies. Our solutions reveal that the marginal tax rate is a function with an order greater than one.

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1 Introduction

How important are tax benefits from debt? This question was not only the title of a famous paper (Graham (2000)) but also the description of a research program looking into the influence of tax shields on the value of a company. Using formal models this question was raised and answered as early as Modigliani and Miller (1963) using a very simple financing policy (constant debt). Later Miles and Ezzell (1980) were able to give a closed-form solution for another financing policy (constant leverage ratio) that remains one of the most popular assumptions in finance until today. Until then, research moved to empirical and simulation studies.

Both theoretical results above have a common element. In Modigliani-Miller's case the tax benefits are linear in the amount of today's debt D (see below). If the assumptions of Miles-Ezzell are satisfied, the tax benefits are linear in the leverage ratio l as well. If we use the concept of elasticity the immediate result is that the tax benefit has an elasticity of one with respect to debt.

Such results should be empirically observed when debt levels change. And this is the point where the issue gets interesting. Many papers have over and over again argued that the effect of debt on the value of the tax shield is much less than both theories (be it Modigliani-Miller or Miles-Ezzell) predict. Myers et al. (1998) have argued that taxes are of third-order importance in the hierarchy of corporate decisions.

The reason seems intuitively clear. Until now in any model where corporate taxes are introduced gains and losses are treated symmetrically. But if losses are, for example, not taxed at all but gains are subject to tax this will influence the value of the tax shield and hence also the elasticity. We would expect that the value of the tax shield is not a linear function of debt and hence the influence is of order less than one. Up to now this result could only be verified using simulation models or empirical studies, a closed-form solution was out of reach: Particularly worth mentioning are Shevlin (1990), Graham (1996a), Graham (1996b), Graham (2000), Graham (2003), and Graham (2006), Graham and Mills (2008), Graham and Kim (2009), Blouin, Core, and Guay (2010). Koch (2013, Part E) discussed thoroughly the weaknesses of such simulation studies.

This is the point where our paper starts. Our aim is to present a model where gains are taxed differently from losses and we will present a closed-form solution for the value of the tax shield. This closed-form solution clearly shows that the elasticity of the tax shield with respect to debt is clearly lower than one, pointing in the right direction.

In particular, we will look at the so-called marginal corporate tax rate (MTR) of a levered firm. Knowing the higher the debt the higher the firm value, this MTR has been employed in the literature as the term that epitomizes the influence of a corporate tax on firm value. The marginal tax rate concretely measures the increase of the present value of all future tax shields from a marginal rise of the present value of all future incomes, given that the company is unlevered. Hence, we will define formally the MTR as the quotient of the value of the tax shield and the value of the levered firm (see equation (2) below). We will establish under reasonable assumptions closed-form solutions for this MTR.

2 Assumptions

Our considerations are based on a rather ordinary set of premises. The market has the usual properties: Firstly, there is a risk-free asset with interest rate r_f which, for simplicity, is assumed to be constant over time. Also, the market is free of arbitrage and hence there is a risk-neutral probability measure Q such that any claim can be evaluated using the discounted Q-expected cash-flow of that claim.[1]

The firm we want to consider has unlevered pre-tax cash flows CF_t^u that are auto-regressive,

$$CF_t^u = CF_{t-1}^u(1+\varepsilon_t)$$

for all $t > 0$. The random variables ε_t are assumed to be independent and identically distributed (iid), with the expectation of zero. Furthermore, we assume $\varepsilon_t > -1$. Hence the unlevered cash flows cannot grow and will never be negative.[2]

Given all the assumptions above the price V_t^u of an unlevered (post-tax) cash flow stream CF_s^u $(s = t+1, \ldots)$ is given by the sum of its Q-expected and discounted value:

$$V_0^u = \sum_{t=1}^{\infty} \frac{E_Q[(1-\tau)CF_t^u]}{(1+r_f)^t}. \tag{1}$$

Lastly, we assume that the unlevered company posseses capital costs that are constant over time. From this, for the unlevered company we immediately obtain

$$V_t^u = \frac{CF_t^u}{k}.$$

Now, let us introduce debt. The (now levered) company will use an amount of debt at time t. An equation applies to the valuation of this company which is quite similar to equation

[1] This fundamental theorem has been used extensively in option pricing, however, in valuation its use is not very popular. Kruschwitz and Löffler (2006) systematically tried to utilize it for valuation purposes.

[2] The formal details of this approach are developed in depth in Kruschwitz and Löffler (2006, chapter 1).

(1). However, its value V_0^l will be determined by the cash flows of the levered firm. We will focus on two different types of financing policies that play an important role in the theory of business valuation.

Fixed leverage ratios The first financing policy is characterized by the fact that the managers of the company fix deterministic leverage ratios l_t for the future. This is well known in the literature as it is the prerequisite for using WACC in firm valuation, see Miles and Ezzell (1980). Because the future values of the indebted firm V_t^l are stochastic, the same applies for the future amounts of debt, $l_t\, V_t^l = D_t$. For simplicity, assume that the future leverage ratio is constant over time, $l_t = l_0$ $(\forall t > 0)$.

Fixed amounts of debt Following the second financing policy the managers would fix the future amount of debt, D_t, deterministically. For convenience, assume that this amount remains constant over time, $D_t = D_0$ $(\forall t > 0)$. This type of policy was discussed by Modigliani and Miller (1963). Considering again that the future values of the indebted firm are stochastic, then the future debt ratios of the firm must also be stochastic under this financing policy, $l_t = D_0/V_t^l$.

MTR is finally being defined by

$$MTR := 1 - \frac{V_0^u}{V_0^l}. \tag{2}$$

We are interested in closed-form solutions for the MTR, particularly if gains and losses are taxed differently.

3 Main Results

3.1 Financing Policy with Constant Leverage Ratios

We first want to assume that the managers of the firm follow a financing policy with a deterministic and constant leverage ratio, $l_0 = l_1 = \ldots = l$.

It is an easy task to determine the MTR if gains and losses are taxed symmetrically. This case was addressed by Miles and Ezzell (1980). Their result is

$$\left(1 - \frac{1+k}{1+r_f}\,\frac{r_f}{k}\,\tau l\right) V_0^l = V_0^u,$$

where k is the cost of capital of the unlevered company. Obviously, the value of the tax shield is linear in the leverage ratio l, and the elasticity of tax benefit with respect to leverage is one. We get the following result.

Proposition 1 (Symmetric Taxation of Gains and Losses, Miles and Ezzell 1980) *If gains and losses are taxed at a rate of τ then*

$$MTR_{symmetric} = \frac{1+k}{1+r_f}\,\frac{r_f}{k}\,\tau\, l. \tag{3}$$

The derivation of a closed-form equation for the MTR under asymmetric taxation is harder. Assume that losses cannot be imputed at all. Let $WACC$ represent the weighted average cost of capital and $WACC = CF_t^u/V_t^l$ for some t. We get the following result.[3]

Proposition 2 (Asymmetric Taxation of Gains and Losses) *If gains are taxed, while losses are not imputed at all, then*

$$MTR_{asymmetric} = \frac{1+k}{1+r_f}\,\frac{r_f}{k}\,\tau\, l\, f\left(\frac{r_f\, l}{WACC}\right) \tag{4}$$

$f(\cdot)$ is a monotonically decreasing function with values between 0 and 1. $WACC$ is not stochastic and even constant.

Comparing equations (3) and (4) with each other reveals an interesting fact. The MTR differ from each other only by the factor of $f\left(\frac{r_f\, l}{WACC}\right)$ and $0 \le f\left(\frac{r_f\, l}{WACC}\right) \le 1$ must hold.

Considering an example is always enlightening. Let us assume that ε_t regarding Q is uniformly distributed on the interval $[-\frac{1}{2}, \frac{1}{2}]$. Calculating the function $f(\cdot)$ for this case yields

$$f(x) = \begin{cases} 1 & x < \frac{1}{2}, \\ \frac{1}{8}\left(12 - \frac{1}{x} - 4x\right) & \frac{1}{2} \le x < \frac{3}{2}, \\ \frac{1}{x} & \frac{3}{2} \le x. \end{cases}$$

Figure 1 shows the functional relationship between the MTR and the leverage ratio, its main influencing factor.

3.2 Financing Policy with Constant Amounts of Debt

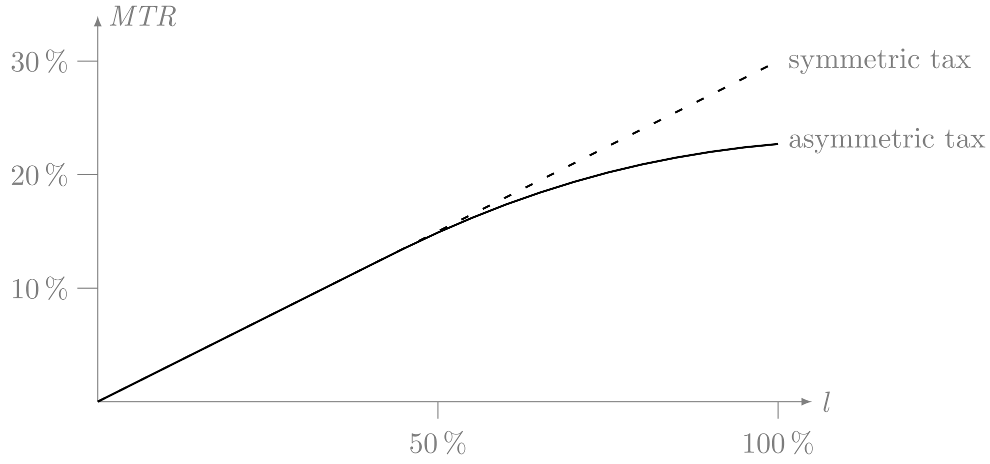
Now assume that the firm follows a financing policy with deterministic and constant amounts of debt, $D_0 = D_1 = \ldots = D$. The future values of the levered firm are stochastic. Hence, due to $l_t := D/V_t^l$ the future leverage ratios are stochastic as well. By contrast, the current leverage ratio of the former section was a number.

Under symmetric taxation the value of the levered firm at each time is

$$V_t^l = V_t^u + \tau\, D\,,$$

[3] We have moved the proof to the appendix.

Figure 1: MTR under constant leverage ratios ($k = 10\,\%$, $r_f = 10\,\%$, $\tau = 30\,\%$) with ε_t regarding Q being uniformly distributed on $[-\frac{1}{2}, \frac{1}{2}]$



From this, immediately

$$\frac{V_t^l - V_t^u}{V_t^l} = \frac{\tau\, D}{V_t^l} = \tau\, l_t\,.$$

These terms are stochastic for any $t > 0$. Only the current MTR (i.e., at $t = 0$) is deterministic.

Proposition 3 (Symmetric Taxation of Gains and Losses, Modigliani and Miller 1963) *Under symmetric taxation of gains and losses the MTR at time $t = 0$ is deterministic and is described as*

$$MTR_{symmetric} = \tau\, l_0\,. \tag{5}$$

The result is different if gains and losses are taxed differently.[4]

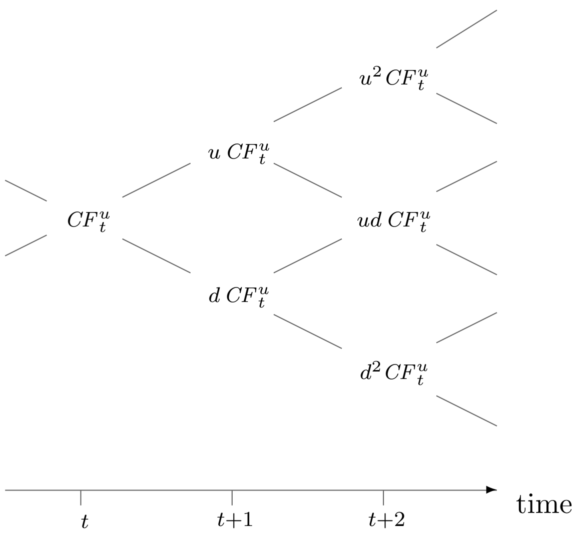
Proposition 4 (Asymmetric Taxation of Gains and Losses) *If gains are taxed, while losses are not imputed at all, then the MTR at time $t = 0$ is deterministic. Depending on the extent of debt, MTR attains a value between τl_0 and $\frac{\tau}{1+\tau}$. The larger the amount of debt, the greater the MTR.*

The first value τl_0 materializes if D is sufficiently small. The second value $\frac{\tau}{1+\tau}$ results if $l_0 = \frac{1}{1+\tau}$ is achieved. We have no closed-form solution for the MTR if D yields results that are located between τl_0 and $\frac{\tau}{1+\tau}$.

[4] Again, the proof is in the appendix.

We are unable to present a closed-form solution for amounts of debt whose MTR are located between τl_0 and $\frac{\tau}{1+\tau}$. Thus, for a certain interval of debt there is no choice but to proceed as follows: Calculate the levered and unlevered values of the firm based on assumptions about the probability distribution of cash flows, the cost of capital, the tax rate, and the extent of debt. Knowing these values for the relevant combination of parameters, the MTR may finally be determined by employing equation (2).

Figure 2: Binomial tree of cash flows



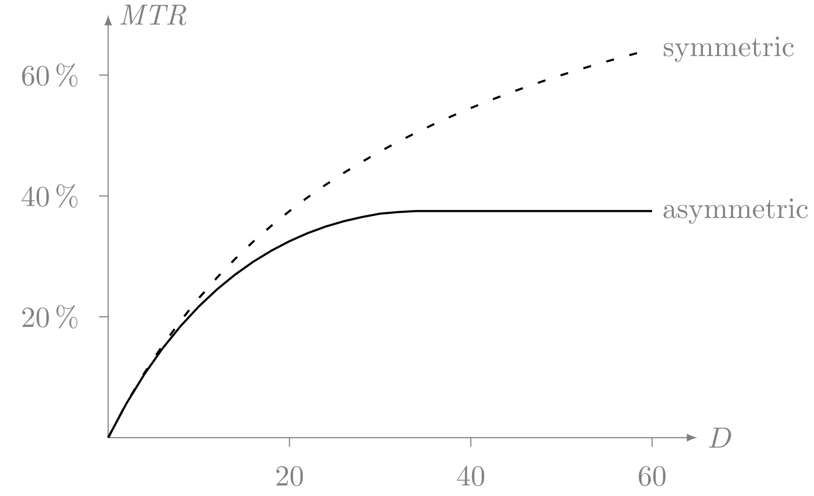
The cash flows follow a binomial tree as shown in Figure 2, with the start value $CF_0^u = 1$ and the growth factors $u = 1.0$ and $d = 0.9$. The risk-neutral probabilities can be determined via option pricing theory using both the cost of capital and the risk-free rate.[5] Calculating the MTR under these conditions gives the result as shown in Figure 3. It is clear that beyond a certain amount of debt the MTR no longer increases, because the resulting losses can not be offset against tax any longer.

4 Conclusion

Evaluating companies requires a lot of information, including the value of the firm's MTR. As a rule, it may be assumed that gains and losses are not taxed identically. In the past 25 years, there have been articles on the estimation of MTR under asymmetric taxation. All papers published so far are working with empirical methods and simulation studies. This

[5]See Kruschwitz and Löffler (2006, p. 42f.).

Figure 3: MTR under constant amounts of debt ($k = 5\,\%$, $r_f = 3\,\%$, $\tau = 60\,\%$, $CF_0^u = 1$), when cash flows follow a binomial tree with $u = 1.0$ and $d = 0.9$



paper is the first to attempt an analytical determination of the MTR. Figure 4 summarizes

Figure 4: Marginal tax rates under symmetric and asymmetric taxation

Financing policy	Losses are taxed	Losses are tax free
Fixed leverage ratios	$\frac{1+k}{1+r_f}\,\frac{r_f}{k}\,\tau l$	$\frac{1+k}{1+r_f}\,\frac{r_f}{k}\,\tau l\, f\left(\frac{r_f\, l}{WACC}\right)$
Fixed amounts of debt	τl_0	$\tau l_0 \to \frac{\tau}{1+\tau}$

our findings. This approach may be particularly relevant as applied to business valuation. Solutions could be developed for two popular financing policies.

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5 Appendix

5.1 Proof of proposition 2

First, the unlevered company has after-tax cash flows of $(1-\tau)\widetilde{CF}_t^u$. The levered company can deduct taxes if there are no losses. Hence, its after-tax cash flow is [6]

$$\widetilde{CF}_t^l = \widetilde{CF}_t^u - \tau\big(\widetilde{CF}_t^u - r_f D_{t-1}\big)^+.$$

This gives a tax shield at time t of

$$\begin{aligned} \widetilde{TS}_t &:= \widetilde{CF}_t^u - \tau\big(\widetilde{CF}_t^u - r_f D_{t-1}\big)^+ - (1-\tau)\widetilde{CF}_t^u \\ &= \begin{cases} \tau r_f D_{t-1} & \text{if } \widetilde{CF}_t^u > r_f D_{t-1} \\ \tau \widetilde{CF}_t^u & \text{else.} \end{cases} \\ &= \tau \min(\widetilde{CF}_t^u, r_f D_{t-1}). \end{aligned} \tag{6}$$

From this, using equation (1), the value of the levered company is

$$\widetilde{V}_t^l = \sum_{s=t+1}^{\infty} \frac{E_Q\left[(1-\tau)\widetilde{CF}_s^u + \tau \min(\widetilde{CF}_s^u, r_f l \widetilde{V}_{s-1}^l)|\mathcal{F}_t\right]}{(1+r_f)^{s-t}}$$

or by employing the stochastic and time-dependent variable

$$\widetilde{WACC}_s := \frac{\widetilde{CF}_s^u}{\widetilde{V}_s^l} \tag{7}$$

$$\widetilde{V}_t^l = \sum_{s=t+1}^{\infty} \frac{E_Q\left[(1-\tau)\widetilde{CF}_s^u + \tau \min(\widetilde{CF}_s^u, \frac{r_f\, l}{\widetilde{WACC}_{s-1}} \widetilde{CF}_{s-1}^u)|\mathcal{F}_t\right]}{(1+r_f)^{s-t}}$$

It follows from Kruschwitz and Löffler (2013, proposition 2) that there must be a unique solution. However, it is not obvious how to determine that solution. Claiming that

$$WACC = \frac{\widetilde{CF}_s^u}{\widetilde{V}_s^l} \tag{8}$$

is deterministic and constant will prove to be correct. From our first assumption we get

$$\widetilde{CF}_s^u = \widetilde{CF}_{s-1}^u\,(1+\tilde{\varepsilon}_s)$$

[6] The symbol X^+ means $\max(X, 0)$.

for an iid variable ε_s. Using equation (8), insertion yields

$$\begin{aligned} V_t^l &= V_t^u + \sum_{s=t+1}^{\infty} \frac{E_Q\left[\tau \min(CF_{s-1}^u(1+\varepsilon_s), \frac{r_f\, l}{WACC}\; CF_{s-1}^u) \mid \mathcal{F}_t\right]}{(1+r_f)^{s-t}} \\ &= V_t^u + \tau \sum_{s=t+1}^{\infty} \frac{E_Q\left[CF_{s-1}^u \min\left(1+\varepsilon_s, \frac{r_f\, l}{WACC}\right) \mid \mathcal{F}_t\right]}{(1+r_f)^{s-t}}\,. \end{aligned} \tag{9}$$

The random variables

$$CF_{s-1}^u = CF_0^u(1+\varepsilon_1)(1+\varepsilon_2)\cdots(1+\varepsilon_{s-1})$$

and

$$\min\left(1+\varepsilon_s, \frac{r_f\, l}{WACC}\right)$$

are independent of each other. Under this condition, the expectation of the product equals the product of the expectations. Hence using $x := (r_f\, l)/WACC$ yields

$$\begin{aligned} V_t^l &= V_t^u + \tau \sum_{s=t+1}^{\infty} \frac{E_Q\left[CF_{s-1}^u \mid \mathcal{F}_t\right]\; E_Q\left[\min\left(1+\varepsilon_s, \frac{r_f\, l}{WACC}\right) \mid \mathcal{F}_t\right]}{(1+r_f)^{s-t}} \\ &= V_t^u + \tau \sum_{s=t+1}^{\infty} \frac{E_Q\left[CF_{s-1}^u \mid \mathcal{F}_t\right]\; \frac{r_f\, l}{WACC}\, E_Q\left[\min\left(\frac{1+\varepsilon_s}{x}, 1\right) \mid \mathcal{F}_t\right]}{(1+r_f)^{s-t}}\,. \end{aligned} \tag{10}$$

We now focus on a function

$$f(x) =_{\text{Def}} E_Q\left[\min\left(\frac{1+\varepsilon_t}{x}, 1\right) |\mathcal{F}_s\right], \qquad t > s.$$

for $x > 0$. This function is dependent on three terms, namely x, the information $\mathcal{F}_s$, and the random variable ε_t. The latter being iid, this is an unconditional expectation that depends only on x. Therefore

$$f(x) = E_Q\left[\min\left(\frac{1+\varepsilon_t}{x}, 1\right)\right]$$

must hold. Now it can easily be shown that when x is small,

$$\lim_{x\to 0} f(x) = E_Q\left[\min\left(\lim_{x\to 0} \frac{1+\varepsilon_t}{x}, 1\right)\right] = 1\,,$$

because $\varepsilon_t > -1$, and when x is large

$$\lim_{x\to \infty} f(x) = E_Q\left[\min\left(\lim_{x\to \infty} \frac{1+\varepsilon_t}{x}, 1\right)\right] = 0\,.$$

The function is monotonically decreasing with x.

We can now determine the tax shield using the newly defined function $f\left(\frac{r_f\,l}{WACC}\right)$. Inserting the term into equation (10) yields

$$\begin{aligned} V_t^l &= V_t^u + \frac{r_f\,l}{WACC}\,\tau \sum_{s=t+1}^{\infty} \frac{E_Q\left[CF_{s-1}^u \,|\, \mathcal{F}_t\right]\; E_Q\left[\min\left(\frac{(1+\varepsilon_s)\,WACC}{r_f\,l},1\right)\right]}{(1+r_f)^{s-t}} \\ &= V_t^u + \frac{r_f\,l}{WACC}\,\tau\, f\left(\frac{r_f\,l}{WACC}\right) \sum_{s=t+1}^{\infty} \frac{E_Q\left[CF_{s-1}^u \,|\, \mathcal{F}_t\right]}{(1+r_f)^{s-t}} \\ &= V_t^u + \frac{r_f\,l}{WACC}\,\tau\, f\left(\frac{r_f\,l}{WACC}\right) \frac{CF_t^u + V_t^u}{1+r_f}\,. \end{aligned} \tag{11}$$

This is a closed-form equation for the tax shield.

This result is based on the mere assumption of $WACC$ being deterministic and constant. If we can trust this result, our assumption was justified. We have to show that if there is a constant and deterministic $WACC$, there is a unique solution. To this end, insert the capital costs equations into equation (11):

$$\frac{CF_t^u}{WACC} = \frac{CF_t^u}{k} + \frac{r_f\,l}{WACC\,(1+r_f)}\,\tau\, f\left(\frac{r_f\,l}{WACC}\right)\left(1+\frac{1}{k}\right) CF_t^u\,.$$

This can easily be transformed to

$$WACC = k - \frac{1+k}{1+r_f} r_f\,\tau\, l\, f\left(\frac{r_f\,l}{WACC}\right)\,.$$

This corresponds to the adjustment formula of Miles and Ezzell (1980) except for the term $f(\cdot)$.

To assure ourselves that a unique solution exists for $WACC$, consider two cases. For $WACC \to 0$ the left-hand side (LHS) of the equation goes to zero, while the right-hand side (RHS) goes to $k > 0$. So the RHS is larger than the LHS. Assuming, however, that $WACC \to \infty$, the LHS goes beyond all limits and is positive, while the RHS remains finite. Because of the monotonicity of the function there can be only one unique solution for $f(\cdot)$. The MTR results easily from equation (11):

$$\begin{aligned} MTR_{\text{asymmetric}} &= 1 - \frac{\frac{CF_t^u}{k}}{\frac{CF_t^u}{WACC}} = 1 - \frac{WACC}{k} \\ &= \frac{1+k}{1+r_f}\frac{r_f}{k}\,\tau\, l\, f\left(\frac{r_f\,l}{WACC}\right)\,. \end{aligned}$$

This completes the proof.

5.2 Proof of proposition 4

Recall equation (6)

$$TS_t = \tau \min(CF_t^u, r_f D_{t-1}).$$

From this, for the levered firm with constant amounts of debt

$$V_0^l = V_0^u + \tau \sum_{t=1}^{\infty} \frac{E_Q[\min(CF_t^u, r_f D)]}{(1+r_f)^t}.$$

Obviously, we must now distinguish two cases. If $r_f D \leq CF_t^u$ ("sufficiently small amount of debt"), it is the known case

$$V_0^l = V_0^u + \tau D$$

and therefore, as with symmetric taxation

$$MTR_{\text{asymmetric}}^{\text{case 1}} = \tau l_0. \tag{12}$$

However, if $r_f D > CF_t^u$ ("sufficiently large amount of debt"), then

$$V_0^l = V_0^u + \tau \sum_{t=1}^{\infty} \frac{E_Q[CF_t^u]}{(1+r_f)^t} = (1+\tau) V_0^u$$

applies. From this follows directly

$$MTR_{\text{asymmetric}}^{\text{case 2}} = \frac{\tau}{1+\tau}. \tag{13}$$

Note that $l_0 \geq 0$ must be provided. Hence, for sufficiently small D the MTR may be vanishingly small, but can never become negative. For sufficiently large debt, the MTR is positive and independent of the extent of debt. As a result, we can generally realize that

$$MTR_{\text{asymmetric}} \leq \tau \min\left(l_0, \frac{1}{1+\tau}\right) \tag{14}$$

must hold. Furthermore, MTR is a continuous function in D. Consequently, for increasing D, the marginal tax rate must grow from τl_0 to $\frac{\tau}{1+\tau}$.

This completes the proof.

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