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Abstract

The Debt Tax Shield, Economic Growth and Inequality

We study the implications of the corporate debt tax shield in a growth economy that taxes household income and firm profits and redistributes tax revenues in an attempt to harmonize the lifetime consumption opportunities of households that differ in their endowments. Our model predicts that the debt tax shield (1) increases the risk-free rate, (2) leads to a higher growth rate of the economy, and (3) increases the degree of disparity in households' lifetime consumption opportunities. We further show that the debt tax shield affects the tradeoff between the goals of achieving a high growth rate of the economy and a low degree of inequality and quantify this tradeoff.

Key Words: debt tax shield, macroeconomic growth, redistributive tax system
JEL Classification Codes: E21, E23, G11, H23, H31, H32

1 Introduction

Departing from the pioneering work of Modigliani and Miller (1958), a huge literature investigates corporate capital structure decisions. Especially the tax-deductibility of corporate interest expenses – the corporate debt tax shield – has caught a lot of attention in both theoretical and empirical work. This literature demonstrates that the debt tax shield heavily affects corporate financial structure decisions. However, the macroeconomic implications of the debt tax shield, which our work focuses on, have been largely overlooked so far.

We set up a general-equilibrium model with a representative firm, households that differ by their initial endowments, and a government that taxes household income as well as firm profits and redistributes tax revenues in an attempt to reduce disparities in lifetime consumption opportunities among households. Households earn income by investing into risky corporate equity, risk-free corporate debt, and risk-free bonds traded among the households.

Our model makes two main predictions. First, it predicts the debt tax shield to increase the risk-free rate, reflecting that debt holders want their share of the tax advantage from the debt tax shield. Second, the higher risk-free rate decreases the price of future over present consumption, which leads to higher savings and thus, in the end, to a higher growth rate of the economy.

On the other hand, a partial effect of the debt tax shield is to lower tax revenues, thereby lowering transfers paid out to poorer households which, ultimately, leads to a higher degree of inequality in households' lifetime consumption opportunities. In this manner, a high growth rate of the economy and a reduction of the disparities in lifetime consumption opportunities among households trade off against each other. For example, a reduction in the gross growth rate of consumption by 0.7% can increase the consumption share of a poorer household with an initial endowment of 20% of the average initial endowment from 25.2% of the average consumption level to 29.8%.

An endogenous determination of corporate capital structure is important for a better understanding of the macroeconomic implications of the debt tax shield. In our model, we operate with a representative firm for which corporate leverage is chosen such that the payout to shareholders is non-negative in all states of the world. Although any single firm may be subject to default risk, this is reasonably assumed not to be the case for the representative firm, which is assumed to maximize shareholder value by choosing a maximal degree of levering subject to this constraint. That is, if the tax burden on firm profits paid out to households as interest is lower than that paid out as dividend, there is a tax advantage to debt financing, and the firm operates with leverage;

Otherwise the firm remains unlevered. Whether such a tax advantage exists depends, among other things, on whether the debt tax shield applies. Hence, the decision of the firm on whether to lever up or not is affected by whether a debt tax shield exists or not.

Our work contributes to two important lines of literature. It complements the literature dealing with the macroeconomic implications of taxes. It is a well-known fact that it is generally not optimal to tax accumulating production factors, because this discourages savings, slows down factor accumulation and thus, ultimately, innovation (Mukherjee, Singh, and Zaldokas, 2016) and economic growth (e.g., Diamond, 1975; Eaton and Rosen, 1980; Varian, 1980; Judd, 1985; Chamley, 1986; Jones, Manuelli, and Rossi, 1997).[1] Simultaneously, Hackbarth, Miao, and Morellec (2006) and Chen (2010), among others, demonstrate that macroeconomic conditions affect corporate capital structure decisions. However, the reverse channel, i.e., how the debt tax shield and its effect on corporate capital structure decisions affect macroeconomic conditions, such as the risk-free rate, the growth rate of the economy, or the allocation of resources among households, has received surprisingly little attention so far. Simultaneously, our work extends this line of literature by allowing for non-uniform taxation of capital income.

Our work further contributes to a growing literature on the implications of the debt tax shield. The debt tax shield has recently caught renewed interest in both theoretical and empirical work. Empirical work estimates that the debt tax shield accounts for about 10% of corporate values (e.g., Graham, 2000; Kemsley and Nissim, 2002; vanBinsbergen, Graham, and Yang, 2010), depicts the evolution of corporate leverage ratios over time (DeAngelo and Roll, 2015; Graham, Leary, and Roberts, 2015) as well as over the business cycle (Korajczyk and Levy, 2003; Halling, Yu, and Zechner, 2016), and documents that taxes in general, and the debt tax shield in particular, significantly affect corporate capital structure decisions (e.g., MacKie-Mason, 1990; Graham, 1996, 1999; Gordon and Lee, 2001; Hovakimian, Opler, and Sheridan, 2001; Bell and Jenkinson, 2002; Graham and Lucker, 2006; Becker, Jacob, and Jacob, 2013; Longstaff and Strebulaev, 2014; Devereux, Maffini, and Xing, 2015; Doidge and Dyck, 2015; Faccio and Xu, 2015; Heider and Ljungqvist, 2015; Faulkender and Smith, 2016; Ljungqvist, Zhang, and Zuo, 2017). Schepens (2016) argues that this makes tax shields a valuable tool for policy makers. However, all these papers focus on the impact of the debt tax shield for corporate valuation and capital structure decisions, but do not investigate the broader macroeconomic implications of the debt tax shield, which are the focus of our work.

Theoretical work, including Miles and Ezzell (1980) and Cooper and Nyborg (2006), has so far primarily focused on the valuation of the debt tax shield. A notable exemption is the

[1] Boskin (1978), Blanchard and Perotti (2002), Romer and Romer (2010), and Cloyne (2013) provide supporting empirical evidence. Optimal redistribution is studied, among others, in Golosov, Troshkin, and Tsyvinski (2016).

work of Fischer and Jensen (2016) that investigates how the debt tax shield affects households' consumption-investment strategies via the government's budget constraint. However, their work builds on an endowment economy model. In their framework, the growth rate of the economy is thus exogenously given, which renders an investigation of the broader macroeconomic implications impossible. Our work instead builds on a production economy and predicts that the debt tax shield significantly affects macroeconomic variables, such as the risk-free rate, inequality, and the growth rate of the economy.

This paper contributes to a line of research that Fama (2011) calls one of the big open challenges in financial economics: understanding the implications of corporate taxation. Our work extends the existing literature in several important dimensions. It shows that the debt tax shield not only affects corporate leverage, but also has broader macroeconomic implications. The debt tax shield has an increasing effect on the risk-free rate, which in turn decreases the relative price of future versus present consumption. As a result, savings and investments increase, resulting in a higher growth rate of the economy.

The debt tax shield also leads to lower tax revenues, thereby to lower transfers from richer to poorer households and thus, ultimately, to a higher degree of inequality in households' lifetime consumption opportunities. That is, the debt tax shield contributes to a higher growth rate of the economy at the expense of a higher degree of inequality. More generally, the objectives of attaining a high growth rate of the economy and a reduction of the degree of disparity in households' lifetime consumption opportunities trade off against each other.

Intuitively, the government could try to increase the growth rate of the economy by investing into the production process itself. However, households can and will undo any effect that such government policy may be intended to have. Hence, there is no scope for such fiscal interventions in our model.

This paper proceeds as follows. Section 2 outlines our model. In section 3, we present its analytical solution and discuss our model's predictions. In section 4, we illustrate the quantitative implications of the debt tax shield for the risk-free rate, economic growth, and inequality. Section 5 concludes. The appendix provides proofs of our theorems.

2 The debt tax shield in a production economy

2.1 The economy

We consider an economy populated by n households and a representative firm, that makes up the production sector. The firm has a risky one-period production technology. That is, it only generates an output in the next period. This output can either be consumed or be reinvested in preparation for consumption in the subsequent periods. The output produced and available at time t depends on the evolution of the economy. It is given by $G_t I^a_{t-1}$, where G_t is the gross growth factor per unit of investment made at time $t-1$ and I^a_{t-1} denotes the aggregate investment in the production technology. For simplicity, we assume that the growth rates are independent and identically distributed copies of a discrete random variable G with M possible realizations G_m, where $G_1 > G_2 > \cdots > G_M$. In the sequel we assume that these M realizations have equal probabilities $1/M$.[2]

Our production technology model is a discrete time version of the classical Cox-Ingersoll-Ross model (Cox, Ingersoll, and Ross, 1985), without intertemporal uncertainty about the production technology. The production technology can be thought of as a farmer growing a perishable output, such as corn, with an identical distribution of the outcome from year to year. The next year's harvest depends on how much of this year's harvest is used for replanting and on the exogenously given realization of the growth rate of the output process, that may, e.g., reflect different weather conditions.

2.2 Corporate leverage

To run the production technology, the firm issues equity and corporate debt that the households can invest into. The aggregate investment, I^a_t, made at time t is financed by the aggregate amount of equity invested, E^a_t, and the aggregate amount of corporate debt, δ^a_t, outstanding from time t to $t+1$. An endogenous determination of corporate leverage in response to tax incentives is a centerpiece in understanding the macroeconomic implications of the debt tax shield. We follow one of the standard assumptions in the literature and assume that the firm's CEO chooses a constant leverage ratio L. Apart from this constraint on the relation between E^a_t and δ^a_t, the supply of aggregate investment opportunities is perfectly elastic. The use of a constant leverage ratio is often referred to as the Miles-Ezzell assumption (Miles and Ezzell, 1980). It is used, among others, in the work of Cooper and Nyborg (2006, 2008).

If the total tax burden on firm profits paid out to households as interest on corporate debt is higher than that paid out as dividend to equity holders, i.e., there is a tax advantage of equity financing, the CEO decides to remain unlevered. Otherwise, the CEO chooses the maximum possible degree of leverage that ensures a non-negative return to shareholders in all states.[3] The non-negativity of net returns simultaneously ensures a positive tax basis from

[2] The assumption of equal probabilities is solely made to ease notation. Our results can be generalized to allow for unequal probabilities with similar qualitative conclusions, although with a significantly blown-up amount of notation.

[3] In the proof of Theorem 1, we show that this is tantamount to maximizing the expected gross growth on investments into firm equity under the risk-neutral measure.

taxing corporate profits. Since the total tax burden on firm profits paid out to households as interest depends on whether the debt tax shield applies, corporate leverage is affected by whether the debt tax shield applies.

2.3 Traded assets

Households can trade three assets. First, households can trade a locally risk-free one-period bond paying a pre-tax return of r_t from time t to $t+1$. We denote household j's position in that asset by $\beta_{t,j}$. This asset comes in zero net supply. That is, if some households want to hold a long position in that asset, the market equilibrium has to bring about an interest rate that makes other households willing to issue such an asset. Second, households can invest into the firm's equity that entitles them to the firm's payout in proportion to their share of equity. We denote household j's investment into the firm's equity from time t to $t+1$ by $E_{t,j}$ and its share of equity by $\alpha_{t,j} = \frac{E_{t,j}}{E_t^a}$. Third, households can invest into one-period corporate bonds, issued by the firm. We denote household j's position in corporate bonds from time t to $t+1$ by $\delta_{t,j}$. Since the firm only issues bonds up to a limit where the net return on equity is non-negative, corporate bonds are default-free, therefore perfect substitutes for the risk-free bond traded among households, and thus bear the same yield. The households' initial endowments are denoted by $W_{0,j} > 0$ and their shares of the total initial endowment $W_0^a = \sum_{j=1}^n W_{0,j}$ are denoted by $\alpha_{0-,j} = \frac{W_{0,j}}{W_0^a} > 0$.

2.4 The redistributive tax system

Throughout the last centuries, most industrial nations around the world implemented tax-financed social insurance and income support programs for poorer households to reduce disparities in lifetime consumption opportunities. Romer and Romer (2014) provide an overview over changes in social security benefits in the United States.

We consider a government that wants to reduce the disparity in lifetime consumption opportunities across households that differ by their initial financial endowments. To attain its goal, the government taxes corporate profits at rate τ_C, households' gains from investments into firm equity at rate τ_E, and households' interest income at rate τ_B. The government implements a linear redistributive tax system from which each household receives an identical share of tax revenues. That is, poorer households pay less in taxes than they receive in transfer income. These households are therefore net recipients of transfer income. Linear redistributive tax systems are commonly used in the public finance literature. Their use ranges back to the work of Romer (1975) and Meltzer and Richard (1981) and has later been used, among others, in Alesina and Angeletos (2005), Sialm (2006), Fischer and Jensen

(2015), and Pástor and Veronesi (2016, 2017).

The redistribution mechanism implies that the government neither builds up wealth nor debt. Within the time horizon of our model, any government debt must be settled through tax payments by the households.[4] Consequently, government debt would never be considered net wealth by the households, cf. also the reasoning in Barro's seminal work (Barro, 1974). We provide a more formal argument that there is no room for an active fiscal policy in our model in section 3.3.

2.5 The debt tax shield

Debt tax shields for corporate interest expenses exist in many countries to avoid a double-taxation of interest at both the company level and the level of the final recipient of the interest payment. He and Matvos (2016) even suggest subsidizing debt in industries with socially wasteful competition to prepone firm exits. Whether a debt tax shield exists or not directly affects corporate capital structure decisions, because the debt tax shield reduces the after-tax cost of debt, thus making debt-financed investments more desirable.[5] The debt tax shield also directly affects the payout, P_t, to equity holders:

$$P_t = I_{t-1}^a \left(1 + g_t \left(1 - \tau_C\right)\right) - \delta_{t-1}^a \widehat{R}_{t-1} \tag{1}$$

at time t, in which $g_t = G_t - 1$ is the net growth rate of investments into the firm's production technology, $\widehat{R}_{t-1} = 1 + r_{t-1}\left(1 - \widehat{\tau}_C\right)$ is the firm's gross after-tax risk-free rate after accounting for whether the debt tax shield exists or not, and

$$\widehat{\tau}_C = \begin{cases} \tau_C & \text{(with the debt tax shield)} \\ 0 & \text{(without the debt tax shield)} \end{cases} \tag{2}$$

is the tax rate applicable to the firm's interest payments.[6] With the constant leverage ratio, $L = \delta_{t-1}^a / E_{t-1}^a$, the payout to shareholders from Equation (1) can be rewritten as

$$P_t = E_{t-1}^a \left(1 + L\right)\left(1 + g_t \left(1 - \tau_C\right)\right) - E_{t-1}^a L \widehat{R}_{t-1}. \tag{3}$$

[4] We disregards the possibility that the government can embark on a Ponzi scheme and can ignore its long-run budget constraint.

[5] We assume throughout that the representative firm operates with corporate debt when the debt tax shield applies. In the proof of Theorem 1, we show that this is the case, if the after tax return on equity income is lower than the after tax return on interest income. Without corporate debt, the debt tax shield obviously has no effect.

[6] We do not explicitly regard the case, where interest expenses are deductible, but the tax compensation for deductions is lower than the tax paid on corporate profits ($0 < \widehat{\tau}_C < \tau_C$), throughout. Our model can be readily applied to these cases.

When the debt tax shield applies, the firm faces lower debt servicing costs, implying a higher amount remaining for its shareholders.

2.6 The household optimization problem

Each household maximizes its present discounted utility from consumption over an N-period investment horizon subject to its intertemporal budget constraint. Households have a common utility discount factor ρ and a time-additive constant relative risk aversion (CRRA) utility function with risk aversion parameter $\gamma \geq 0$. That is, the utility from a consumption of C is given by:

$$U(C) = \begin{cases} \frac{C^{1-\gamma}}{1-\gamma} & \text{if } \gamma \neq 1 \\ \ln(C) & \text{if } \gamma = 1. \end{cases} \tag{4}$$

The evolution of household j's wealth after accounting for taxes consists of three components. First, the household receives the payout from its equity investments. After accounting for taxation on the household level, this leaves the household with an income of

$$\alpha_{t-1,j}\left(\left(P_t - E^a_{t-1}\right)(1-\tau_E) + E^a_{t-1}\right). \tag{5}$$

Second, the household receives income from its holdings of the risk-free asset and corporate debt of

$$\left(\beta_{t-1,j} + \delta_{t-1,j}\right)\widetilde{R}_{t-1}, \tag{6}$$

where

$$\widetilde{R}_{t-1} = 1 + r_{t-1}(1-\tau_B) \tag{7}$$

is the gross risk-free rate from time $t-1$ to t after accounting for taxation on the household level. Third, the household receives transfer income, the level of which depends on the government's tax revenues that in turn consist of three components. First, the government generates a tax revenue of $\tau_E\left(P_t - E^a_{t-1}\right)$ by taxing gains from equity investments. Second, the taxation of interest on the household level provides a tax revenue of $\tau_B r_{t-1}\delta^a_{t-1}$. Finally, the government taxes the firm profit, Ω_t:

$$\Omega_t = \begin{cases} I^a_{t-1}g_t - r_{t-1}\delta^a_{t-1} & \text{(with the debt tax shield)} \\ I^a_{t-1}g_t & \text{(without the debt tax shield)} \end{cases} \tag{8}$$

at the corporate tax rate τ_C. Total tax revenues are thus given by

$$\tau_E\left(P_t - E^a_{t-1}\right) + \tau_B r_{t-1}\delta^a_{t-1} + \tau_C\Omega_t, \tag{9}$$

of which each household receives an equal share. From Equations (8) and (9), tax revenues are lower with the debt tax shield than without. The evolution of household j's wealth is given by (XYZ Substitute for $+\tau_C\Omega_t$)

$$\begin{aligned} W_{t,j} =& \alpha_{t-1,j}\left(\left(P_t - E^a_{t-1}\right)(1-\tau_E) + E^a_{t-1}\right) + \left(\beta_{t-1,j} + \delta_{t-1,j}\right)\widetilde{R}_{t-1} + \\ & \frac{1}{n}\left(\tau_E\left(P_t - E^a_{t-1}\right) + \left(\tau_B - \widehat{\tau}_C\right) r_{t-1}\delta^a_{t-1} + \tau_C I^a_{t-1} g_t\right). \end{aligned} \tag{10}$$

We denote the effective rate of (double) taxation that the equity return is subject to by $\widetilde{\tau}$:

$$\widetilde{\tau} \equiv \tau_C + \tau_E(1-\tau_C) = \tau_E + \tau_C(1-\tau_E) \;\Leftrightarrow\; 1-\widetilde{\tau} \equiv (1-\tau_E)(1-\tau_C). \tag{11}$$

Household j's optimization problem is then given by:

$$\max_{\{C_{t,j},E_{t,j},\delta_{t,j},\beta_{t,j}\}_{t=0}^{t=N}} U\left(C_{0,j}\right) + \sum_{t=1}^{N} \rho^t \mathbb{E}_0\left[U\left(C_{t,j}\right)\right]$$

$$\text{s.t. } W_{t,j} = C_{t,j} + E_{t,j} + \beta_{t,j} + \delta_{t,j}, \quad t = 0,1,2,\ldots,N \tag{12}$$

$$E_{N,j} = \delta_{N,t} = \beta_{N,j} = 0. \tag{13}$$

Table 1 summarizes the notation used in this paper. Having presented our model, we next turn to its closed-form solution and show how the debt tax shield affects the economy.

3 Implications of the debt tax shield

In this section, we present the general-equilibrium solution to the model introduced in section 2 in closed form. To ensure a non-negative tax base, we impose an upper bound on the degree of corporate levering:

$$L \leq \frac{g_M}{\left(\bar{g} - g_M\right)} \frac{1-\tau_B}{\left(1-\widehat{\tau}_C\right)\left(1-\tau_E\right)}, \tag{14}$$

where $g_M = G_M - 1$ is the lowest possible net growth rate of the production technology, and $\bar{g}$ is the expected value of the net growth rate g under the risk-neutral measure. To ensure a positive upper bound on the level of corporate leverage, we assume $g_M > 0$ and provide a formal derivation in the proof of Theorem 1 that this constraint not only ensures a positive upper bound on corporate leverage, but simultaneously also guarantees a non-negative tax base.

Table 1
Definition of variables

Variable	Description
ρ	The households' common utility discount factor
γ	The households' common degree of relative risk aversion
$\alpha_{0-,j}$	Household j's initial endowment
$\alpha_{t,j}$	Household j's share of equity investments in the production process from time t to time $t+1$
$\beta_{t,j}$	Number of risk-free assets, issued by households, that is held by household j from time t to $t+1$
$\delta_{t,j}$	Number corporate bonds held by household j from time t to $t+1$
$E_{t,j}$	Household j's equity investment from time t to time $t+1$
δ_t^a	Number of corporate bonds outstanding from time t to $t+1$
E_t^a	Aggregate equity investment from time t to time $t+1$
I_t^a	Total investment in production process from time t to $t+1$, $I_t^a = E_t^a + \delta_t^a$
L	Firm's constant leverage ratio: $L = \delta_t^a / E_t^a$
$C_{t,j}$	Household j's consumption at time t
C_t^a	Aggregate consumption at time t
τ_E	Tax rate applicable to household income from equity
τ_B	Tax rate applicable to household income from bonds
τ_C	Corporate tax rate
$\widehat{\tau}_C$	Corporate tax rate, applicable to a firm's interest payments
$\widetilde{\tau}$	Total tax rate applicable to a household's equity income: $\widetilde{\tau} = \tau_C + \tau_E(1-\tau_C)$
$\widehat{\tau}$	Tax rate measuring the loss in tax revenues from corporate and equity taxation per unit of equity replaced with debt: $= \widehat{\tau}_C + \tau_E(1-\widehat{\tau}_C)$
$\xi,\ \psi$	The relative tax disadvantage of using equity:$=(1-\tau_E)(1-\widehat{\tau}_C)/(1-\tau_B)$
R_t	Gross risk-free rate before taxes from time t to $t+1$
r_t	Net risk-free rate before taxes from time t to $t+1$: $r_t = R_t - 1$
$\widetilde{R}_t$	Gross risk-free rate after taxes on household level from time t to $t+1$
$\widehat{R}_t$	Gross risk-free rate after taxes on corporate level from time t to $t+1$
O_t	Output at time t
Ω_t	Taxable corporate income at time t
P_t	Payout from the firm to equity holders at time t
G_t	Gross growth factor of output O from time $t-1$ to t, $G_t = O_t/O_{t-1}$
G	Version of the independent stochastic gross growth factors G_t
$\{G_j\}_{j=1}^{j=M}$	Outcomes of G: $G_1, G_2, \ldots, G_M$
g_t	Net growth factor of output O from time $t-1$ to t, $g_t = G_t - 1$
$W_{t,j}$	Household j's wealth level at time t, before consumption and investment
n	Number of households in the economy
N	Length of investment horizon in periods

3.1 Macroeconomic effects

We begin the presentation of our results by turning to the implications of the debt tax shield for the risk-free rate and growth of the economy in Theorem 1:

Theorem 1. *For the risk-free rate and the rate of economic growth it holds that:*

1. *The risk-free rate r is constant and given by*

$$r = \frac{\bar{g}\,(1-\tau_C)}{\frac{1}{1+L}\frac{1-\tau_B}{1-\tau_E} + \frac{L}{1+L}\,(1-\widehat{\tau}_C)} = \frac{\bar{g}\xi}{\frac{1}{1+L} + \frac{L}{1+L}\psi}, \tag{15}$$

where

$$\xi \equiv \frac{(1-\tau_E)(1-\tau_C)}{1-\tau_B} = \frac{1-\widetilde{\tau}}{1-\tau_B}, \quad \psi \equiv \frac{(1-\tau_E)(1-\widehat{\tau}_C)}{1-\tau_B} = \frac{1-\widehat{\tau}}{1-\tau_B}. \tag{16}$$

ξ is a measure of the tax burden on equity relative to debt financing when the debt tax shield applies,[7] *in which case $\xi = \psi$. If the tax shield does not apply, $\psi \equiv \frac{1-\tau_E}{1-\tau_B}$ and ψ is then similarly the measure of the tax burden on equity relative to debt financing.*

If there is a tax advantage of using equity, then the firm remains unlevered ($L = 0$), $\xi > 1$, and the risk-free rate becomes:

$$r = \bar{g}\xi. \tag{17}$$

If there exists a tax advantage to debt, the interest rate is increasing in the degree of leverage L. If the firm optimizes its leverage ratio by setting L equal to the right hand side of Equation (14) and the interest rate becomes:

$$r = \begin{cases} \bar{g}\xi + g_M(1-\xi) & \textit{(with debt tax shield)} \\ \bar{g}\xi + g_M(1-\xi-\tau_C) & \textit{(without debt tax shield).} \end{cases} \tag{18}$$

2. *Aggregate consumption, C_t^a, and total investments, I_t^a into the real investment opportunity are given by*

$$C_t^a = (1-F_t)\,W_t^a, \quad I_t^a = F_t W_t^a, \tag{19}$$

where F_t is the fraction of total output, $W_t^a = I_{t-1}^a G_t$, that is invested into the real investment opportunity as either equity or debt. F_t is state independent, decreasing

[7] Cf. Miller (1977).

over time, and can be expressed in explicit form as:

$$F_t = \begin{cases} \frac{1-H^{N-t}}{1-H^{N-t+1}} & \textit{for } H \neq 1 \\ \frac{N-t}{N-t+1} & \textit{for } H = 1, \end{cases} \tag{20}$$

where

$$H = \left(\frac{\rho}{M} \sum_{m=1}^{M} G_m^{-\gamma} \right)^{-\frac{1}{\gamma}} \widetilde{R}^{-\frac{1}{\gamma}}. \tag{21}$$

F_t is higher when the debt tax shield applies. For $N \to \infty$ it holds that:

$$\lim_{N \to \infty} F_t = \begin{cases} 1 & \textit{for } H \leq 1 \\ \frac{1}{H} & \textit{for } H > 1. \end{cases} \tag{22}$$

When the debt tax shield applies, the share of wealth invested, F_t, as well as the the utility from aggregate consumption is higher.

3. *The growth rate of consumption is the same for all households and follows the i.i.d. process with distribution*

$$\frac{C_{t+1}^a}{C_t^a} = \frac{C_{t+1,j}}{C_{t,j}} = \frac{1}{H} G. \tag{23}$$

4. *In explicit form, the total tax revenue, $TTR_t = \tau_E \left(P_t - E_{t-1}^a \right) + (\tau_B - \widehat{\tau}_C) r_{t-1} \delta_{t-1}^a + \tau_C I_{t-1}^a g_t$, can be written as*

$$TTR_t = I_{t-1}^a \cdot \begin{cases} \widetilde{\tau} g_t - g_M (1 - \tau_B)(1 - \xi) & \textit{(leverage and tax shield)} \\ \widetilde{\tau} g_t - g_M (1 - \tau_B)(1 - \xi - \tau_C) & \textit{(leverage and no tax shield)} \\ \widetilde{\tau} g_t & \textit{(no leverage)}, \end{cases} \tag{24}$$

where I_{t-1}^a can also be expressed as

$$I_{t-1}^a = W_0^a \cdot \left(\prod_{i=1}^{t-1} G_i \right) \cdot \left(\prod_{i=0}^{t-1} F_i \right) = W_0^a \cdot \left(\prod_{i=1}^{t-1} G_i \right) \cdot \frac{1 - H^{N+1-t}}{1 - H^{N+1}}. \tag{25}$$

Proof *The proof of Theorem 1 is provided in Appendix A.*

Theorem 1 reveals how tax rates and the debt tax shield affect macroeconomic variables, such as the risk-free rate, aggregate investments and economic growth, as well as tax revenues.

Even though the debt tax shield only directly affects the corporate tax basis of a levered firm, item 1 relevals that in general equilibrium, it also affects the risk-free rate. From

Equations (16) and (17), it holds that $r\,(1-\tau_B) = \bar{g}\,(1-\widetilde{\tau})$ when the firm does not lever up. In equilibrium, the return on the risk-free asset after taxes then corresponds to the expected after-tax return on equity under the risk-neutral measure.

When there is a tax advantage to corporate debt, the firm levers up. From Equation (18), the risk-free rate then contains an additional term, which takes a positive value. Corporate levering increases the risk-free rate, because the tax advantage to corporate debt increases the desirability of investing into corporate equity. To raise the desired amount of corporate debt, the firm has to offer a higher interest rate. The effect of corporate levering on the interest rate is stronger when the debt tax shield applies. In that case, the firm faces a lower corporate tax burden. The lower tax burden increases the after-tax profit and makes equity investments even more desirable. To make households willing to nevertheless purchase the amount of corporate bonds, the firm wants to issue, it has to offer a higher risk-free rate than in the absence of the debt tax shield.

Item 2 reveals two important properties about the fraction F_t of aggregate wealth invested. First, F_t is state independent, reflecting the i.i.d. growth rates of the production process. Second, F_t is positively related to the interest rate. The only endogenous effect on H, and thereby also on F_t, is through the interest rate. From Equations (20) and (21), an increase in the interest rate decreases the parameter H, which in turn increases F_t. Economic growth thus increases in the risk-free rate, reflecting that a higher risk-free rate decreases the price of future relative to present consumption.

Third, F_t is higher when the debt tax shield applies. Again, the channel driving this result is the impact of the debt tax shield on the risk-free rate. In the presence of the debt tax shield, the aggregate share of wealth consumed is lower and the share invested is higher. The higher share of wealth invested ultimately leads to more factor accumulation and thus, a higher growth rate of the economy. The higher growth rate of the economy leads to higher welfare level from aggregate consumption, i.e., to a higher welfare for a representative investor.[8]

In addition to the well-documented effects of the debt tax shield on corporate financial structure, our model shows that in equilibrium, the debt tax shield also has important macroeconomic effects. In particular, the debt tax shield increases the risk-free rate, the growth rate of the economy, and aggregate welfare. Our model predicts a positive relationship between the risk-free rate and economic growth. This result is in direct contrast to the popular view that investment increases when the risk-free rate decreases. In our model, economic growth increases with the risk-free rate, because a high risk-free rate decreases the

[8] The expression for utility from aggregate consumption and the proof of this statement is given in Appendix A.

price of future versus present consumption and thus increases household savings, which in turn has a positive impact on economic growth.

From item 4, tax revenues at time t depend on aggregate investments, I_{t-1}^a, in the previous period, as well as on whether the firm operates with leverage and on whether the debt tax shield applies. When the firm operates without leverage, tax revenues at time t per unit of aggregate investment at time $t-1$ stem from the taxation of firm profits on the corporate and household level. In particular, tax revenues are independent of whether the debt tax shield applies, because without corporate debt, there are no corporate interest expenses that would be subject to the debt tax shield. When the firm operates with leverage, it does so because of a tax advantage to corporate debt. The second term in the upper two cases in Equation (24) measures the implied loss in tax revenues. It is higher when the debt tax shield applies. Hence, the share of wealth redistributed is lower when the debt tax shield applies. The absolute amount of tax revenues and redistributions, however, may be higher, reflecting that aggregate investments are higher when the debt tax shield applies.

From items 1, 2, and 4, we can conclude that the two objectives of attaining a high rate of growth of the economy and a reduction in the disparities in lifetime consumption opportunities among households trade off against each other. To reduce disparities in lifetime consumption opportunities, the government has to increase transfer payments to poorer households. That is, it has to increase tax revenues by increasing tax rates or remove an existing debt tax shield. From Equation (15), these policies lead to a reduction in the risk-free rate. The reduction in the risk-free rate in turn reduces F_t, the share of wealth invested, by increasing the parameter H from Equation (21). A reduction in disparities in lifetime consumption opportunities among households thus comes at the cost of reducing economic growth. In section 4.2 we investigate in more detail how reductions in lifetime consumption opportunities trade off against macroeconomic growth. In particular, we document how this tradeoff is quantitatively affected by the existence or absence of the debt tax shield.

From item 3, the growth rate of consumption is identical among households. Households establish a linear risk sharing rule via their trading of financial assets. The attempt to establish such a linear sharing rule has important implications for households' consumption-investment strategies and the effectiveness of fiscal policy that we turn to in sections 3.2 and 3.3.

3.2 Households' consumption-investment policies

Having derived closed-form solutions for the risk-free rate and economic growth, we next show how the debt tax shield affects individual households' consumption and investment strategies. Our key findings are summarized in Theorem 2:

Theorem 2. *For household j's consumption and investment policies, it holds that:*

1. *The allocation of macroeconomic risk is in accordance with a linear sharing rule relative to the distribution of wealth after taxes. Household j's position in the risk-free asset from time t to t + 1 is proportional to the aggregate investment, I_t^a, and given by:*

$$\frac{\beta_{t,j} + \delta_{t,j}}{I_t^a} = \alpha_{t,j} \frac{L}{1+L} + \frac{1}{\widetilde{R}} \cdot \left(\alpha_{t,j} - \frac{1}{n} \right) \left(\frac{L}{1+L} r \left(\tau_B - \widehat{\tau} \right) - \widetilde{\tau} \right). \tag{26}$$

For $N \to \infty$, the position in the risk-free asset is given by the expression in Equation (26) with $\alpha_{t,j}$ substituted by the the limiting value of the equity position, α_j, from Equation (33).

2. *Household j's consumption share, $\omega_j \equiv C_{0,j}/C_0^a$, is constant over time and fulfills:*

$$\omega_j - \frac{1}{n} = D \left(\alpha_{0-,j} - \frac{1}{n} \right), \tag{27}$$

where

$$D = \frac{H - Y}{H^{N+1} - Y^{N+1}} \frac{H^{N+1} - 1}{H - 1} \tag{28}$$

is a measure of the degree of disparity in lifetime consumption opportunities and

$$Y = \frac{(\widetilde{R} - \widetilde{\tau}) + \frac{L}{1+L} r \left(\tau_B - \widehat{\tau} \right)}{(1 - \widetilde{\tau}) \widetilde{R}}. \tag{29}$$

It holds that

$$\lim_{N \to \infty} D = \frac{H - Y}{H - 1}. \tag{30}$$

3. *Household j's equity share, $\alpha_{t,j}$, is given by*

$$\alpha_{t,j} = \frac{1}{n} + \frac{\left(\omega_j - \frac{1}{n} \right)}{1 - \widetilde{\tau}} Z_t \tag{31}$$

$$Z_t = \frac{H^{N-t} - Y^{N-t}}{H - Y} \frac{(H - 1)}{H^{N-t} - 1}. \tag{32}$$

Poorer households' equity shares increase over time and richer households' decrease. It holds that $D = 1/Z_{-1}$. For $N \to \infty$, the equity share is a constant:

$$\alpha_j = \frac{1}{n} + \left(\alpha_{0-,j} - \frac{1}{n} \right) \frac{1}{1 - \widetilde{\tau}}. \tag{33}$$

Proof *A detailed proof for all items of Theorem 2 is given in Appendix B.*

Theorem 2 shows that the debt tax shield not only affects the risk-free rate and economic growth, but also the degree of harmonization in lifetime consumption opportunities as well as individual households' consumption and investment strategies.

Theorem 2, item 1 reveals how households choose their exposure to the risk-free asset. From Equation (26), household j's position in the risk-free asset consists of two terms. The first term, $\alpha_{t,j}\frac{L}{1+L}$ is zero, when the firm operates without debt and is proportional to the share of corporate debt of the firm's total capital otherwise. When the firm operates with corporate debt, its shareholders have an implicit short position in the risk-free asset. For a household j that holds a share of $\alpha_{t,j}$ of firm equity, the implicit short position in the risk-free asset is $\alpha_{t,j}\frac{L}{1+L}I_t^a$. Hence, household j needs a position of $\alpha_{t,j}\frac{L}{1+L}I_t^a$ in the risk-free asset to undo this implicit short position. This hedging demand simultaneously ensures that aggregate demand for the risk-free asset meets aggregate supply.

In the second term, $\frac{1}{\tilde{R}}\left(\alpha_{t,j}-\frac{1}{n}\right)\left(\frac{L}{1+L}r\left(\tau_B-\widehat{\tau}\right)-\widetilde{\tau}\right)$, the factor $\alpha_{t,j}-\frac{1}{n}$ is negative for poorer households with below-average equity holdings, $\alpha_{t,j}<\frac{1}{n}$. When the firm operates without leverage, the second term is proportional to $\widetilde{\tau}$, the effective tax rate applicable to returns from investments into firm equity. From Equation (24), transfer income is then proportional to the tax revenues from taxing firm profits at rate $\widetilde{\tau}$. Given that returns to equity are subject to macroeconomic risk, the transfer income of poorer households with below-average equity holdings is subject to macroeconomic risk; i.e., these households have implicit long positions in firm equity, which they react to by decreasing their investments into firm equity and increasing their investments into the risk-free asset.

When the firm operates with corporate leverage, the term $\frac{L}{1+L}r\left(\tau_B-\widehat{\tau}\right)$ may become nonzero. It measures how the government's tax revenues are affected by corporate debt. For every unit of corporate debt that replaces firm equity, the government collects an additional tax revenue of $r\tau_B$ from the taxation of the return on the risk-free asset, but loses a tax revenue of $r\widehat{\tau}$ from the taxation of the replaced firm equity. When $\tau_B<\widehat{\tau}$, tax revenues decrease with corporate levering. Poorer households with below-average equity exposures react to this implied reduction in their risk-free transfer income by increasing their exposure to the risk-free asset. With tax-neutrality between corporate debt and equity, i.e., when the Miller (1977) conditions hold and $\tau_B=\widehat{\tau}$, tax revenues are independent of the level of corporate levering and households do not have to adjust their portfolio positions to changes in their transfer income. Because $\widehat{\tau}\geq\widetilde{\tau}$, there exists a tax advantage to equity for $\tau_B>\widehat{\tau}$. The firm then operates without corporate leverage, and $L=0$. In sum, the term $\frac{L}{1+L}r\left(\tau_B-\widehat{\tau}\right)$ is only nonzero when $\tau_B<\widehat{\tau}$.

Theorem 2, item 2 reveals that the linear sharing rule from item 1 implies households attain time- and state-independent constant consumption shares. From Equation (27), household j's deviation from an equal consumption share is proportional to its initial deviation from the average initial endowment. The proportionality factor D can be interpreted as an inequality measure and take values between 0 and 1. A value of $D = 1$ represents no reduction in the disparity in lifetime consumption opportunities where each households' consumption share corresponds to its initial endowment. It corresponds to a world with no taxation at any level and, consequently, no redistribution. $D = 0$ implies the largest possible degree in the reduction of disparity in lifetime consumption opportunities where all households, irrespective of their initial endowments, attain the same consumption share. Because such a perfect harmonization of lifetime consumption opportunities is only achievable with an infinite investment horizon and tax rates of 100%, a zero value of D is only a theoretical lower bound.

From Theorem 1, we know that the two goals of achieving a high level of macroeconomic growth and a reduction in the disparities in lifetime consumption opportunities among households trade off against each other. High levels of the growth scaling parameter $1/H$ tend to occur simultaneously with high levels of the disparity measure D. We quantify this trade-off in more detail in section 4.2.

Theorem 2, item 3 shows that with a finite investment horizon, households' equity exposures are converging towards each other over time. Poorer households' equity exposures increase and richer households' decrease, reflecting that poorer households need to build up savings to finance their consumption share.

3.3 Fiscal policy

Our results in Theorem 1 show that removing an existing debt tax shield decreases aggregate production and thus the growth rate of the economy. In this section, we show why the government cannot reestablish the same level of aggregate production as in the presence of the debt tax shield via an active fiscal policy. Intuitively, the government could try to compensate for the decrease in aggregate production by investing in the production technology itself, thus financing this investment by issuing government debt. Government bonds and privately issued bonds are both risk-free assets and are therefore perfect substitutes carrying the same interest rate. To avoid lengthy notation, we assume that government bonds are single-period bonds. Hence, we do not need to introduce further variables into the model. Instead, we require that $\sum_{j=1}^{n} \beta_{t,j} = \beta_t^a$, where β_t^a is the total amount of government bonds outstanding from time t to $t+1$. If the government invests into firm equity, household j's evolution of wealth is given by:

$$
\begin{aligned}
W_{t,j} = & \left(\left(E_{t-1,j} - \frac{E^a_{t-1}}{n} \right) (1 - \tau_E) + \frac{E^a_{t-1}}{n} \right) \left((1 + L) (1 + g_t (1 - \tau_C)) - L \widehat{R}_{t-1} \right) + \\
& \tau_E \left(E_{t-1,j} - \frac{E^a_{t-1}}{n} \right) + \frac{\tau_C}{n} E^a_{t-1} (1 + L) g_t + (\beta_{t-1,j} + \delta_{t-1,j}) \widetilde{R}_{t-1} + \\
& \frac{E^a_{t-1}}{n} L r_{t-1} (\tau_B - \widehat{\tau}_C) + \frac{1}{n} \beta^a_{t-1} \left(G - \widetilde{R}_{t-1} \right).
\end{aligned} \tag{34}
$$

Compared to the evolution of wealth without government debt from Equation (10), Equation (34) contains the additional term $\frac{1}{n}\beta^a_{t-1}\left(G - \widetilde{R}_{t-1}\right)$ that, ceteris paribus, leads to a higher effective exposure to the real investment and a lower exposure to the risk-free asset for every household j.

In our model, where markets are complete and households have rational expectations, households can and will undo any effect fiscal policy might be intended to have. Irrespective of whether the government implements an active fiscal policy or not, households strive for a linear risk sharing rule. Recalling that the leverage ratio L from Equation (14) is given in terms of the input parameters of the model, any deviation from its optimal level can and will be corrected by the supply and demand decisions of the firm and the households.

When the government purchases firm equity, from Equation (34), every household j's exposure to firm equity increases by $\frac{\beta^a_{t-1}}{n}$, implying an increase in every household's exposure to macroeconomic risk. Households react to this increase by reducing their equity holdings by $\frac{\beta^a_{t-1}}{n}$ units each. As a consequence, the aggregate demand for firm equity is unchanged, and the corporate leverage ratio is not altered. The government's intervention simultaneously implies a reduction in every household j's effective exposure to the risk-free asset by $\frac{\beta^a_{t-1}}{n}$ units. To undo this government-intervention-implied reduction, every household increases its exposure to the risk-free asset by $\frac{\beta^a_{t-1}}{n}$ units by purchasing government bonds. As a result of the households' adjustments to their trading strategies, the budget equation is fulfilled, households do not alter their consumption plans, they reestablish the linear sharing rule, markets clear, corporate leverage does not change, and the level of real investment remains unchanged. Hence, there is no room for an active fiscal policy.

If the government decides to invest into the firm via corporate debt, household j's budget constraint from Equation (10) is not affected. For every unit of corporate interest income the government earns, it has to pay exactly the same amount to the government debt holders. The firm reacts to the deviation from its optimal leverage ratio by decreasing its amount of corporate bounds outstanding to households by β^a_{t-1} units. In sum, each household reduces its corporate bond holdings by $\frac{\beta^a_{t-1}}{n}$ units and increases its government bond holdings by $\frac{\beta^a_{t-1}}{n}$ units. As a result of the firm's and the households' adjustments to their trading strategies, the budget equation is fulfilled, households do not alter their consumption plans, they reestablish the linear sharing rule, markets clear, corporate leverage does not change, and the level of real investment remains unchanged. In other words, there is perfect crowding out of the government intervention and no room for an active fiscal policy in our model. This model prediction is in line with the empirical evidence in Graham, Leary, and Roberts (2014) and Demirici, Huang, and Sialm (2017) that government debt crowds out corporate debt. Again, there is no room for an active fiscal policy in our model.

4 Quantitative effects

In this section we illustrate the quantitative implications of the debt tax shield for the risk-free rate, the growth rate of the economy, and household consumption. We want to illustrate both immediate and long-term consequences of the debt tax shield. We therefore choose an investment horizon of $N = 100$ periods and assume one period to correspond to one year. The degree of risk aversion and the households' time preference parameter are set to $\gamma = 1$ and $\rho = 0.98$, which is in the range of values typically considered in the literature.[9] The tax rates are set to $\tau_E = 20\%$, $\tau_B = 39.6\%$, and $\tau_C = 35\%$, the current top tax rates for U.S. households and corporations.

For simplicity, we focus on a setting with $M = 2$ possible realizations throughout our numerical analysis. We set the mean of the growth rate of our real investment opportunity to 3.1%, corresponding to the average real post-war GDP growth in the US. The standard deviation of the real investment opportunity's growth rate is chosen to attain a level of corporate levering that is in line with the historical empirical evidence. More specifically, we set the standard deviation to 1.8%, implying a debt-to-capital ratio of 46%, which is in the range of historical ratios reported by Graham, Leary, and Roberts (2015).[10]

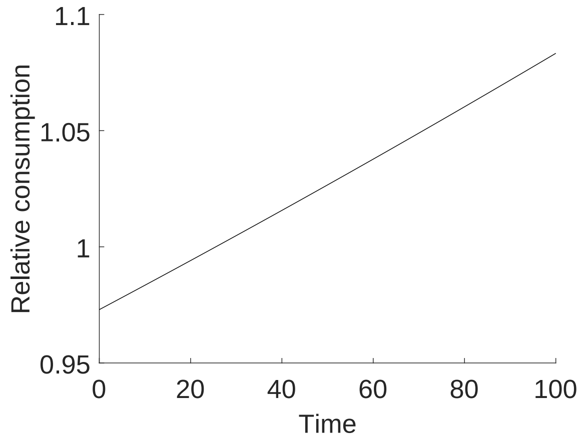
4.1 Risk-free rate, macroeconomic growth, and consumption

From Theorem 1, the debt tax shield increases the risk-free rate, increases economic growth, and alters the intertemporal allocation of consumption. In this section we quantify the order of magnitude of these effects. We begin the discussion of our results with the risk-free rate that drives both economic growth (Equations (20) and (21)) and the strength of

[9] We also explored the robustness of our results to other choices of γ and ρ. These changes only affect our results quantitatively, but not qualitatively, and are therefore not presented here. They are, however, available from the authors upon request.

[10] We also explored other choices of the distribution of the growth rate of the real investment opportunity. Given that these changes only affect our results quantitatively, but not qualitatively, they are not reported here, but available from the authors upon request.

Figure 1
Impact of debt tax shield on consumption



This figure depicts the evolution of consumption over time in a setting with the debt tax shield relative to a setting without the debt tax shield.

the harmonization of lifetime consumption opportunities (Equation (27)). In our base case parameter setting, the risk-free rate is 2.8% when the debt tax shield applies and 2.6% when it does not apply. That is, the debt tax shield increases the risk-free rate by seven percent or 20 basis points.

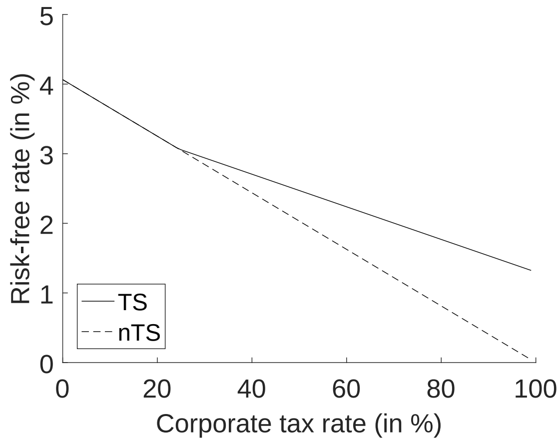
From Theorem 1, we know that the higher risk-free rate increases savings, reflecting the decrease in the price of future relative to present consumption. The higher savings rate alters the intertemporal allocation of consumption and increases the growth rate of the economy. We quantify these effects in Figure 1. The figure depicts the evolution of consumption in a setting with the debt tax shield relative to a setting without over time.

In our base case parameter setting aggregate consumption at time $t = 0$ is 2.7% lower when the debt tax shield applies (Figure 1). That is, the debt tax shield significantly reduces immediate consumption. In the long run, however, the higher savings rate causes a wealth effect that results in the consumption level being higher from time $t = 20$ onwards. At time $t = 100$, consumption in the setting with the debt tax shield is 8.3% higher than in the setting without.

Having shown that whether the debt tax shield applies or not affects the risk-free rate and alters the intertemporal allocation of consumption, we next turn to a demonstration of how the level of the tax burden quantitatively affects our results. For that purpose, we vary the corporate tax rate between $\tau_C = 0\%$ and $\tau_C = 99\%$.[11] We first turn to showing how the

[11] We also explored varying τ_E and τ_B, which resulted in similar results. The results mainly channel them-

Figure 2
Impact of corporate tax rate on risk-free rate



This figure depicts the evolution the risk-free rate in a setting with the debt tax shield (solid line, TS) and without the debt tax shield (dashed line, nTS).

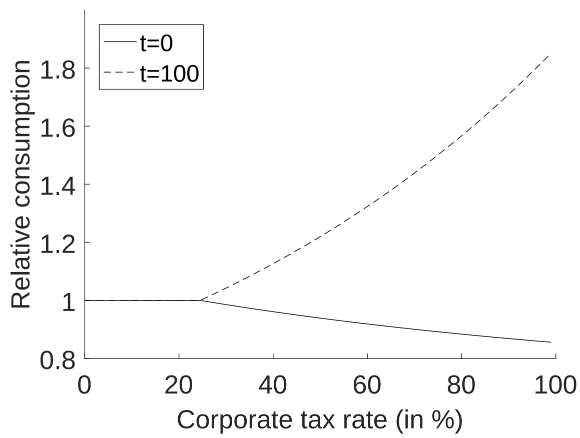
corporate tax rate affects the risk-free rate. Figure 2 depicts the evolution of the risk-free rate in a setting with the debt tax shield (solid line) and without (dashed line).

Confirming our results from Theorem 1, it shows that the risk-free rate decreases in the level of the corporate tax rate. For corporate tax rates below 24.5% the effect is of the same order of magnitude irrespective of whether the debt tax shield applies or not. This is so, because for levels of the corporate tax rate below 24.5%, the firm operates without corporate leverage, irrespective of whether the debt tax shield applies or not. When the firm operates without corporate leverage, it does not matter whether the debt tax shield applies or not, and results do not differ between the two settings.

For levels of the corporate tax rate exceeding 24.5%, it becomes optimal to operate with corporate debt when the tax shield applies, and it remains optimal to be unlevered without the debt tax shield, which results in the linear relationship between the corporate tax rate and the risk-free rate when the debt tax shield does not apply. With the debt tax shield, there is a kink at a corporate tax rate of 24.5%. The debt tax shield reduces the after-tax cost of corporate debt, which makes investments into equity more desirable. The order of magnitude increases with the level of the corporate tax rate. To nevertheless find investors that are willing to hold corporate debt, the firm has to offer a higher risk-free rate when the debt tax shield applies. For example, when corporate taxes do not apply, i.e., for $\tau_C = 0\%$, the risk-free rate is 4.1%. It decreases to 2.5% with the debt tax shield and 2.1% without

selves through the variation of the parameters ξ and ψ.

Figure 3
Impact of corporate tax rate on consumption



This figure depicts the evolution of consumption in a setting with debt tax shield relative to a setting without debt tax shield as a function of the corporate tax rate. The solid lines show results at time $t = 0$, the dashed lines at time $t = 100$.

the debt tax shield for a corporate tax rate of $\tau_C = 50\%$.

Having depicted how the corporate tax rate affects the risk-free rate, we next ask how it affects aggregate consumption and wealth in the economy. Figure 3 depicts consumption in a setting with debt tax shield relative to a setting without as a function of the corporate tax rate. The solid lines show results at time $t = 0$, the dashed lines at time $t = 100$.

Consistent with our results for the risk-free rate, consumption is identical with and without the debt tax shield for corporate tax rates below 24.5%. For rates exceeding 24.5%, the amount spent on consumption at time $t = 0$ is lower when the debt tax shield applies and the order of magnitude of this effect amplifies in the level of the corporate tax rate, reflecting the decreasing price of future relative to present consumption. For example, for a corporate tax rate of $\tau_C = 35\%$, consumption at time $t = 0$ is 2.7% lower with the debt tax shield. The order of magnitude of this effect increases to 6.1% for a corporate tax rate of $\tau_C = 50\%$.

At time $t = 100$, however, relative consumption levels dramatically increase in the level of the corporate tax rate, reflecting that the effect of more households saving in the presence of the debt tax shield amplifies when the level of the corporate tax rate increases. For example, for a corporate tax rate of $\tau_C = 35\%$, consumption at time $t = 100$ is 8.3% higher with the debt tax shield. The order of magnitude of this effect increases to 22.0% for a corporate tax rate of $\tau_C = 50\%$.

4.2 Tradeoff between growth and inequality

We know from Theorem 1 that the growth rate of aggregate consumption and the disparity in lifetime consumption opportunities among households trade off against each other. In this section, we quantify this tradeoff. From Equation (23) the growth rate of consumption is proportional to $1/H$. We therefore interpret $1/H$ as a scaling factor and a measure for economic growth. From Equation (27), the deviation of each households' consumption share from an equal consumption share is the household's deviation from the average initial endowment times D. From Theorem 2, D can therefore be interpreted as a disparity measure. It can take values between 0 and 1. 1 represents the highest level of disparity, where poorer households' consumption shares correspond to their initial endowments. This situation occurs without taxation and redistribution, i.e., for tax rates of $\tau_B = \tau_C = \tau_E = 0\%$. 0 represents the theoretically lowest possible level of inequality, where all households are endowed with equal consumption shares. In reality, an inequality measure of 0 should not be a reasonable objective from a policy makers perspective, because it comes at the cost of removing households' incentives to invest.[12]

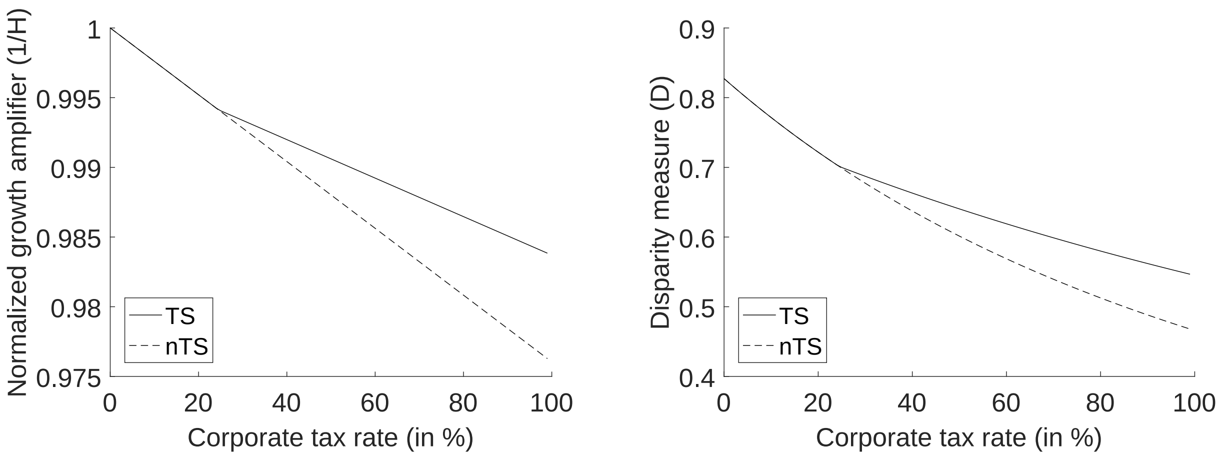
Figure 4 depicts how the corporate tax rate affects economic growth (normalized growth amplifier ($1/H$), left panel) and the disparity measure (D, right panel). We normalize the growth multiplier to 1 for a corporate tax rate of $\tau_C = 0\%$, thus allowing us to easily measure reductions in the annual gross growth rate, G, of the economy relative to a setting with a corporate tax rate of $\tau_C = 0\%$. The solid lines show results when the debt tax shield applies, the dashed lines, when it does not.

Consistent with Theorem 1, Figure 4 shows that higher levels of the corporate tax rate decrease both economic growth and inequality. These effects are amplified in the absence of a debt tax shield. That is, the debt tax shield significantly affects the tradeoff between economic growth and inequality. For example, in our base case parameter setting with the debt tax shield and a corporate tax rate of $\tau_C = 35\%$, the annual gross growth rate of the economy is 0.7% lower than with a corporate tax rate of $\tau_C = 0\%$, and the inequality measure is reduced from 0.827 to 0.675, implying that the consumption share of a poorer household with an initial endowment of 20% of the average initial endowment increases from 25.2% to 29.8% when the debt tax shield applies. When the debt tax shield does not apply, the annual gross growth rate of consumption is 0.8% lower, and the inequality measure is 0.657, implying the consumption share of the poorer household increases to 30.3%.

For higher tax rates, these effects are further amplified. For example, for a corporate tax

[12] In Fischer and Jensen (2016) we have rationalized the linear taxation and redistribution scheme as the solution to an optimization problem with a government objective function for reducing disparity in consumption opportunities combined with friction cost of collecting taxes.

Figure 4
Tradeoff between growth and inequality



This figure depicts the impact of the corporate tax rate on growth (left panel) and inequality (right panel). The growth amplifier $1/H$ is normalized to 1 for a corporate tax rate of $\tau_C = 0\%$. The solid lines show results with the debt tax shield, the dashed lines without.

rate of $\tau_C = 50\%$, the annual growth rate of the economy is 0.9% lower than with a corporate tax rate of $\tau_C = 0\%$ and the inequality measure is reduced to 0.640, implying that the consumption share of the poorer household increases to 30.8% when the debt tax shield applies. When the debt tax shield does not apply, the annual gross growth rate of consumption is 1.2% lower, and the inequality measure is 0.397, implying that the consumption share of the poorer household increases to 32.0%.

Overall, our results in this section depict quantitatively significant effects of the debt tax shield on the intertemporal allocation of resources, the risk-free rate, macroeconomic growth, and inequality in lifetime consumption opportunities among households.

5 Conclusion

This paper studies the implications of the debt tax shield in a growth economy that taxes household income and redistributes tax revenues in an attempt to harmonize lifetime consumption opportunities among households. Our work complements the literature dealing with the effects of taxes by investigating the macroeconomic implications of the debt tax shield. Simultaneously, it contributes to the growing literature on the implications of the debt tax shield. Whereas this literature has focused on the effects for corporate leverage and valuation, our work explores the broader macroeconomic implications of the debt tax shield.

Our general-equilibrium model predicts that – for a given level of the corporate tax rate – the debt tax shield (1) increases the risk-free rate, (2) leads to a higher growth rate of the economy, and (3) increases the degree of disparity in households' lifetime consumption opportunities. The debt tax shield thus contributes to a higher macroeconomic growth rate at the expense of a higher degree of inequality among households in the economy. The goals of achieving a high growth rate of the economy and a low degree of inequality trade off against each other.

Our work can be extended in multiple directions. For example, it would be interesting to not only allow for heterogeneity among households in terms of their initial endowments, but also in terms of their preferences, even though a solution to such a model is probably no longer possible in closed form. It would further be interesting to consider how progressive taxation affects our results. Finally, it would be fruitful to test our model predictions empirically. We leave these interesting extensions for future research.

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Appendix

A Proof of Theorem 1:

We express the equity variables as $E_t^a = \frac{1}{1+L} I_t^a$ and $E_{t,j} = \alpha_{t,j} E_t^a = \alpha_{t,j} \frac{1}{1+L} I_t^a$. Using Equations (3) and (8), we can rewrite Equation (10) as

$$\begin{aligned} W_{t,j} = & \left(\left(E_{t-1,j} - \frac{E_{t-1}^a}{n} \right) (1 - \tau_E) + \frac{E_{t-1}^a}{n} \right) \left((1+L)\left(1 + g_t (1 - \tau_C)\right) - L\widehat{R}_{t-1} \right) + \\ & \tau_E \left(E_{t-1,j} - \frac{E_{t-1}^a}{n} \right) + \frac{\tau_C}{n} E_{t-1}^a (1+L) g_t + \left(\beta_{t-1,j} + \delta_{t-1,j}\right) \widetilde{R}_{t-1} + \\ & \frac{E_{t-1}^a}{n} L r_{t-1} \left(\tau_B - \widehat{\tau}_C\right) \end{aligned} \tag{A.1}$$

Recalling that $\alpha_{N,j} = \beta_{N,j} = \delta_{N,j} = 0$, each household's Lagrangian can be written as

$$L = \sum_{t=0}^{N} \rho^t \mathbb{E}_0 \left[\frac{C_{t,j}^{1-\gamma}}{1-\gamma} \right] - \lambda_{0,j} \left[C_{0,j} - W_{0,j} + E_{0,j} + \beta_{0,j} + \delta_{0,j} \right] -$$
$$\sum_{t=1}^{N} < \lambda_{t,j}, C_{t,j} + E_{t,j} + (\beta_{t,j} + \delta_{t,j}) > +$$
$$\sum_{t=1}^{N} < \lambda_{t,j}, \left(\left(E_{t-1,j} - \frac{E_{t-1}^a}{n} \right) (1 - \tau_E) + \frac{E_{t-1}^a}{n} \right) \left((1+L)(1 + g_t (1 - \tau_C)) - L\widehat{R}_{t-1} \right) > +$$
$$\sum_{t=1}^{N} < \lambda_{t,j}, \tau_E \left(E_{t-1,j} - \frac{E_{t-1}^a}{n} \right) > +$$
$$\sum_{t=1}^{N} < \lambda_{t,j}, \frac{\tau_C}{n} E_{t-1}^a (1+L) g_t + (\beta_{t-1,j} + \delta_{t-1,j}) \widetilde{R}_{t-1} + \frac{E_{t-1}^a}{n} L r_{t-1} (\tau_B - \widehat{\tau}_C) >$$
$$\tag{A.2}$$

where $< .,. >$ is the scalar product over the relevant states.
The first-order condition with respect to $C_{t,j}$ is

$$\left(\frac{\rho}{M} \right)^t C_{t,j}^{-\gamma} = \lambda_{t,j}. \tag{A.3}$$

The first-order condition with respect to $E_{t,j}$ is

$$\lambda_{t,j} = \sum_{m=1}^{M} \lambda_{t+1,j}^m \left[\left((1+L)(1 + g_m (1 - \tau_C)) - L\widehat{R}_t \right) (1 - \tau_E) + \tau_E \right]. \tag{A.4}$$

The first-order condition with respect to $\beta_{t,j} + \delta_{t,j}$ is

$$\lambda_{t,j} = \widetilde{R}_t \sum_{m=1}^{M} \lambda_{t+1,j}^m. \tag{A.5}$$

To continue, observe that the first-order conditions are homogeneous in the following sense: If a given solution, $(\{C_{t,j}\}_{t=0}^{t=N}; \{\beta_{t,j} + \delta_{t,j}\}_{t=0}^{t=N-1}; \{\lambda_{t,j}\}_{t=0}^{t=N})$, satisfies the conditions for a given wealth level $W_{0,j}$, then

$$\left(x\{C_{t,j}\}_{t=0}^{t=N}; x\{\beta_{t,j} + \delta_{t,j}\}_{t=0}^{t=N-1}; x^{-\gamma}\{\lambda_{t,j}\}_{t=0}^{t=N} \right) \tag{A.6}$$

also satisfies the conditions for the wealth level $xW_{0,j}$. This proportionality property tells us that households in an optimal solution settle on a linear sharing rule for aggregate consumption and linear asset demand functions. From Equation (A.3):

$$\frac{\lambda_{t+1,j}}{\lambda_{t,j}} = \frac{\rho}{M}\left(\frac{C_{t+1,j}}{C_{t,j}}\right)^{-\gamma} = \frac{\rho}{M}\left(\frac{C^a_{t+1}}{C^a_t}\right)^{-\gamma}. \tag{A.7}$$

Proof of item 1

From the evolution of aggregate wealth in the economy, it holds that

$$W^a_t = \sum_{j=1}^{n} W_{t,j} = I^a_{t-1} G_t, \tag{A.8}$$

$$C^a_t = \sum_{j=1}^{n} C_{t,j} = (1 - F_t)\, W^a_t = (1 - F_t)\, I^a_{t-1} G_t, \tag{A.9}$$

and

$$I^a_t = F_t W^a_t. \tag{A.10}$$

The aggregate amount invested into equity and corporate debt is given by $E^a_t = \frac{1}{1+L} I^a_t$ and $\delta^a_t = \frac{L}{1+L} I^a_t$, respectively. From Equation (A.9), it follows that

$$\frac{C^a_{t+1}}{C^a_t} = \frac{1 - F_{t+1}}{1 - F_t} \frac{I^a_t G_{t+1}}{I^a_{t-1} G_t} = \frac{1 - F_{t+1}}{1 - F_t} F_t G_{t+1}. \tag{A.11}$$

Since the term in front of G_{t+1} is state independent, the risk-neutral martingale measure is:

$$q_m = \frac{G_m^{-\gamma}}{\sum_{k=1}^{M} G_k^{-\gamma}}. \tag{A.12}$$

From Equations (A.4), (A.5), and (A.12) we have:

$$\widetilde{R}_t = \mathbb{E}^Q\left[\left((1 + L)\,(1 + g\,(1 - \tau_C)) - L\widehat{R}_t\right)(1 - \tau_E) + \tau_E\right] \quad \Leftrightarrow \tag{A.13}$$

$$r_t = \frac{\mathbb{E}^Q\,[g]\,(1 - \tau_C)}{\frac{1}{1+L}\frac{1-\tau_B}{1-\tau_E} + \frac{L}{1+L}\,(1 - \widehat{\tau}_C)}. \tag{A.14}$$

From Equation (A.14), the risk-free rate is time independent. Hence, we can drop the index t, use $\bar{g} = \mathbb{E}^Q\,[g]$ and rewrite the risk-free rate as:

$$r = \frac{\bar{g}\,(1 - \tau_C)}{\frac{1}{1+L}\frac{1-\tau_B}{1-\tau_E} + \frac{L}{1+L}\,(1 - \widehat{\tau}_C)} = \frac{\bar{g}\xi(1 + L)}{1 + L\psi}. \tag{A.15}$$

When $L = 0$, Equation (A.15) simplifies to the result stated in Equation (17). When there is a tax advantage of using debt, i.e., when $\psi < 1$, the interest rate is an increasing function of the degree of leverage L. When the firm uses the maximum possible leverage, i.e., the value of L where the constraint in Equation (14) is fulfilled with equality, it holds that

$$L = \frac{g_M}{(\bar{g} - g_M)} \frac{1 - \tau_B}{1 - \hat{\tau}} = \frac{g_M}{(\bar{g} - g_M)} \frac{1}{\psi} \tag{A.16}$$

$$1 + L = \frac{g_M(1 - \tau_B) + (1 - \hat{\tau})(\bar{g} - g_M)}{(\bar{g} - g_M)(1 - \hat{\tau})} \tag{A.17}$$

$$\frac{1}{1 + L} = \frac{(\bar{g} - g_M)(1 - \hat{\tau})}{g_M(1 - \tau_B) + (1 - \hat{\tau})(\bar{g} - g_M)} \tag{A.18}$$

$$\frac{L}{1 + L} = \frac{g_M(1 - \tau_B)}{g_M(1 - \tau_B) + (1 - \hat{\tau})(\bar{g} - g_M)}. \tag{A.19}$$

Inserting Equations (A.18) and (A.19) into Equation (15), we obtain the results in Equation (18).

Optimal corporate leverage

From Equation (15), the risk-free rate depends on the leverage ratio, L. The realized return on the real investment, cf. Equation (3), can then be rewritten as

$$\frac{P_t}{E^a_{t-1}} = 1 + g(1 - \tau_C) + L(1 - \tau_C)\left(g - \frac{\psi(1 + L)\bar{g}}{1 + L\psi}\right). \tag{A.20}$$

Similarly, the expected return under the risk-neutral measure is

$$\mathbb{E}^Q\left[\frac{P_t}{E^a_{t-1}}\right] = 1 + \bar{g}(1 - \tau_C) + L(1 - \tau_C)\bar{g}\left(1 - \frac{\psi(1 + L)}{1 + L\psi}\right). \tag{A.21}$$

The right hand side is both time independent and scale invariant. Hence, it is a time-consistent objective for the CEO to choose a constant leverage ratio. The first-order condition with respect to L is

$$\begin{aligned} \frac{\partial \mathbb{E}^Q\left[\frac{P_t}{E^a_{t-1}}\right]}{\partial L} &= (1 - \tau_C)\bar{g}\left(1 - \frac{\psi(1 + L)}{1 + L\psi}\right) - L(1 - \tau_C)\bar{g}\frac{\psi(1 + L\psi) - (1 + L)\psi^2}{(1 + L\psi)^2} \\ &= (1 - \tau_C)\bar{g}\frac{1 - \psi}{(1 + L\psi)^2}. \end{aligned} \tag{A.22}$$

This term is positive if there exists a tax advantage to debt and negative if there exists a tax advantage to equity. Hence, when there exists a tax advantage to debt, the firm's

CEO chooses the maximum possible degree of leverage; when there exists a tax advantage to equity, the CEO chooses not to operate with leverage ($L = 0$). From Equation (A.22) this corporate debt policy is equivalent to maximizing the expected gross growth of investments into firm equity under the risk-neutral measure.

Maximum level of corporate leverage

The maximum possible degree of corporate leverage that ensures a non-negative return on equity has to fulfill the constraint

$$g\left(1-\tau_C\right)+L\left(g\left(1-\tau_C\right)-r\left(1-\widehat{\tau}_C\right)\right)\geq 0. \tag{A.23}$$

This inequality has to hold for all possible g. In particular, it has to hold for the lowest possible g, g_M, where the risk of violating the constraint is highest. Hence,

$$\begin{aligned}
&g_M\left(1-\tau_C\right)+L\left(g_M\left(1-\tau_C\right)-r\left(1-\widehat{\tau}_C\right)\right)\geq 0 \;\Leftrightarrow\\
&L\frac{\bar{g}\left(1-\tau_C\right)\left(1-\widehat{\tau}_C\right)}{\frac{1}{1+L}\frac{1-\tau_B}{1-\tau_E}+\frac{L}{1+L}\left(1-\widehat{\tau}_C\right)}\leq g_M\left(1-\tau_C\right)\left(1+L\right)\;\Leftrightarrow\\
&L\bar{g}\left(1-\widehat{\tau}_C\right)\left(1-\tau_E\right)\leq g_M\left(1-\tau_B+L\left(1-\widehat{\tau}_C\right)\left(1-\tau_E\right)\right)\;\Leftrightarrow\\
&L\leq\frac{g_M}{\left(\bar{g}-g_M\right)\psi},
\end{aligned} \tag{A.24}$$

which verifies the condition stated in Equation (14).

Proof of item 2

From Equation (A.4), we get:

$$1=\sum_{m=1}^{M}\frac{\lambda_{t+1,j}^{m}}{\lambda_{t,j}}\left[\left(\left(1+L\right)\left(1+g_m\left(1-\tau_C\right)\right)-L\widehat{R}_t\right)\left(1-\tau_E\right)+\tau_E\right]. \tag{A.25}$$

With Equation (A.7), this can be rewritten to

$$
\begin{aligned}
1 &= \mathbb{E}_t \left[\rho \left(\frac{C^a_{t+1,m}}{C^a_t} \right)^{-\gamma} \left((1+L)\left(1+g_m\left(1-\tau_C\right)\right) - L\widehat{R}_t \right) \left(1-\tau_E\right) + \tau_E \right] \\
&= \frac{\rho}{M} \sum_{m=1}^{M} \frac{\left(\left(1-F_{t+1}\right) I^a_t G_m\right)^{-\gamma}}{\left(C^a_t\right)^{-\gamma}} \cdot \qquad (A.26) \\
&\quad \left[\left((1+L)\left(1+g_m\left(1-\tau_C\right)\right) - L\widehat{R}_t \right) \left(1-\tau_E\right) + \tau_E \right] \quad \Leftrightarrow \\
\left(C^a_t\right)^{-\gamma} &= \frac{\rho}{M} \left(\left(1-F_{t+1}\right) I^a_t\right)^{-\gamma} \cdot \\
&\quad \sum_{m=1}^{M} G_m^{-\gamma} \left[\left((1+L)\left(1+g_m\left(1-\tau_C\right)\right) - L\widehat{R}_t \right) \left(1-\tau_E\right) + \tau_E \right]. \qquad (A.27)
\end{aligned}
$$

This implies

$$
C^a_t = I^a_t \left(1-F_{t+1}\right) \cdot H, \tag{A.28}
$$

where

$$
H = \left(\frac{\rho}{M}\right)^{-\frac{1}{\gamma}} \left(\sum_{m=1}^{M} G_m^{-\gamma} \left(\left((1+L)\left(1+g_m\left(1-\tau_C\right)\right) - L\widehat{R}_t \right) \left(1-\tau_E\right) + \tau_E \right) \right)^{-\frac{1}{\gamma}}. \tag{A.29}
$$

This can be rewritten as

$$
\begin{aligned}
H^{-\gamma} &= \frac{\rho}{M} \sum_{m=1}^{M} G_m^{-\gamma} \left[\left((1+L)\left(1+g_m\left(1-\tau_C\right)\right) - L\widehat{R} \right) \left(1-\tau_E\right) + \tau_E \right] \\
&= \frac{\rho}{M} \sum_{k=1}^{M} G_k^{-\gamma} \sum_{m=1}^{M} \frac{G_m^{-\gamma}}{\sum_{k=1}^{M} G_k^{-\gamma}} \left[\left((1+L)\left(1+g_m\left(1-\tau_C\right)\right) - L\widehat{R} \right) \left(1-\tau_E\right) + \tau_E \right] \\
&= \frac{\rho}{M} \sum_{k=1}^{M} G_k^{-\gamma} \mathbb{E}^Q \left[\left((1+L)\left(1+g\left(1-\tau_C\right)\right) - L\widehat{R} \right) \left(1-\tau_E\right) + \tau_E \right]. \qquad (A.30)
\end{aligned}
$$

Using Equation (A.13), Equation (A.30) becomes:

$$
H^{-\gamma} = \frac{\rho}{M} \sum_{m=1}^{M} G_m^{-\gamma} \widetilde{R}, \tag{A.31}
$$

and thus

$$
H = \left(\frac{\rho}{M} \sum_{m=1}^{M} G_m^{-\gamma} \right)^{-\frac{1}{\gamma}} \widetilde{R}^{-\frac{1}{\gamma}}. \tag{A.32}
$$

It further holds that

$$W_t^a = C_t^a + I_t^a = I_t^a\left(1 - F_{t+1}\right)H + I_t^a = I_t^a\left(\left(1 - F_{t+1}\right)H + 1\right) \;\Rightarrow \tag{A.33}$$

$$I_t^a = \frac{W_t^a}{1 + \left(1 - F_{t+1}\right)H}, \tag{A.34}$$

and, consequently, that F_t follows the backward difference equation

$$F_t = \frac{1}{1 + (1 - F_{t+1})H} \tag{A.35}$$

with boundary condition $F_N = 0$. The solution to Equation (A.35) is:

$$F_t = \begin{cases} \frac{1-H^{N-t}}{1-H^{N-t+1}} & \text{for } H \neq 1 \\ \frac{N-t}{N-t+1} & \text{for } H = 1. \end{cases} \tag{A.36}$$

The fact that F_t decreases over time can be shown directly – for the cases $H < 1$ and $H > 1$, respectively, or by backwards induction, using that that $0 \leq F_t < 1$:

$$F_t = \frac{1}{1 + (1 - F_{t+1})H} > F_{t+1} \;\Leftrightarrow\; 1 > F_{t+1} + (1 - F_{t+1})F_{t+1}H \;\Leftrightarrow\; 1 > F_{t+1}H. \tag{A.37}$$

Since $F_N = 0$ this inequality in trivially true for N. Assume that it is true for $N, N-1, \ldots, t+1$. Then:

$$F_{t+1}H < 1 \;\Leftrightarrow\; (1 - F_{t+1})H > H - 1 \;\Leftrightarrow\; 1 + (1 - F_{t+1})H > H \;\Leftrightarrow$$
$$F_t H = \frac{H}{1 + (1 - F_{t+1})H} < 1. \tag{A.38}$$

which verifies the claim that F_t decreases over time. The limiting behavior for $N \to \infty$, is trivial for $H \leq 1$. For $H > 1$ we have

$$F_t = \frac{1 - H^{N-t}}{1 - H^{N-t+1}} = \frac{H^{-(N-t)} - 1}{H^{-(N-t)} - H} \underset{N\to\infty}{\longrightarrow} \frac{1}{H}, \tag{A.39}$$

which verifies Equation (22).

The debt tax shield increases economic growth

From Equation (21), H is proportional to $\widetilde{R}^{-\frac{1}{\gamma}}$. Using Equation (A.35) we can show by induction that F_t increases when the debt tax shield is introduced. For $t = N - 1$ it holds that

$$F_{N-1} = \frac{1}{1+H}. \tag{A.40}$$

We know that H is lower with the debt tax shield than without. Hence, F_{N-1} is larger with the debt tax shield than without. For the next step, we assume that for $t < N-1$, F_{t+1} is higher when the debt tax shield applies. At time t it holds that

$$F_t = \frac{1}{1+(1-F_{t+1})\,H}. \tag{A.41}$$

H is lower and (by the induction assumption) F_{t+1} is higher with the debt tax shield than without. Hence, the denominator in Equation (A.41) decreases and, consequently, F_t increases as a result of the debt tax shield.

Utility from aggregate consumption is higher with debt tax shield

From Equation (23) (to be proven below) we have that $C^a_{t+1} = C^a_t\,(G_{t+1}/H)$. The entire consumption path can then be expressed in terms of the initial consumption C^a_0:

$$C^a_t = C^a_0 \prod_{j=0}^{t} \frac{G_j}{H} = W^a_0(1-F_0) \prod_{j=0}^{t} \frac{G_j}{H}. \tag{A.42}$$

Hence, by the i.i.d. assumption of G_j we have:

$$\begin{aligned} \frac{1}{1-\gamma}\sum_{t=0}^{N} \rho^t \mathbb{E}_0\left(C^a_t\right)^{1-\gamma} &= \frac{1}{1-\gamma}\sum_{t=0}^{N} \rho^t \left(W^a_0(1-F_0)\right)^{1-\gamma} \mathbb{E}_0\left(\prod_{j=0}^{t}\left[\frac{G_j}{H}\right]^{1-\gamma}\right) \\ &= \frac{1}{1-\gamma}\sum_{t=0}^{N} \rho^t \left(W^a_0(1-F_0)\right)^{1-\gamma} \left(\mathbb{E}_0\prod_{j=0}^{t}\left[\frac{G_j}{H}\right]^{1-\gamma}\right) \\ &= \frac{1}{1-\gamma}\left(W^a_0(1-F_0)\right)^{1-\gamma} \sum_{t=0}^{N}\left(\frac{\rho\mathbb{E}[G^{1-\gamma}]}{H^{1-\gamma}}\right)^t. \end{aligned} \tag{A.43}$$

From the solution for F_t we get:

$$1-F_0 = 1-\frac{1-H^N}{1-H^{N+1}} = \frac{1-H^{N+1}-(1-H^N)}{1-H^{N+1}} = H^N\frac{1-H}{1-H^{N+1}} = \frac{1}{\sum_{j=0}^{N} H^{-j}}, \tag{A.44}$$

which can be inserted into Equation (A.43):

$$\frac{1}{1-\gamma}\sum_{t=0}^{N}\rho^t\mathbb{E}_0\left(C_t^a\right)^{1-\gamma} = \frac{1}{1-\gamma}\left(W_0^a\right)^{1-\gamma}\left(\sum_{j=0}^{N}H^{-j}\right)^{\gamma-1}\sum_{t=0}^{N}\left(\rho\mathbb{E}[G^{1-\gamma}]\right)^t H^{-t(1-\gamma)}. \quad \text{(A.45)}$$

Differentiation after H in the expected utility produces two terms:

$$\begin{aligned}&\frac{\partial}{\partial H}\left(\frac{1}{1-\gamma}\sum_{t=0}^{N}\rho^t\mathbb{E}_0\left(C_t^a\right)^{1-\gamma}\right) = \\ &\left(W_0^a\right)^{1-\gamma}\left(\sum_{j=0}^{N}H^{-j}\right)^{\gamma-2}\sum_{j=0}^{N}jH^{-j-1}\sum_{t=0}^{N}\left(\rho\mathbb{E}[G^{1-\gamma}]\right)^t H^{-t(1-\gamma)} - \\ &\left(W_0^a\right)^{1-\gamma}\left(\sum_{j=0}^{N}H^{-j}\right)^{\gamma-1}\sum_{t=0}^{N}\left(\rho\mathbb{E}[G^{1-\gamma}]\right)^t tH^{-t(1-\gamma)-1}.\end{aligned} \quad \text{(A.46)}$$

The sign of this derivative is negative if

$$\sum_{j=0}^{N} j\frac{H^{-j}}{\sum_{j=0}^{N}H^{-j}} - \sum_{t=0}^{N} t\frac{\left(\frac{H^{1-\gamma}}{\rho\mathbb{E}[G^{1-\gamma}]}\right)^{-t}}{\sum_{t=0}^{N}\left(\frac{H^{1-\gamma}}{\rho\mathbb{E}[G^{1-\gamma}]}\right)^{-t}} \quad \text{(A.47)}$$

is negative. The expression in Equation (A.47) is the difference between two "duration measures" of an annuity; one with the discount factor H and the other with the discount factor $H^{1-\gamma}/\rho\mathbb{E}[G^{1-\gamma}]$. The duration of an annuity is a decreasing function of the discount factor, so the claim that the expected utility of the aggregate consumption stream varies inversely with H is proven if $H > H^{1-\gamma}/\rho\mathbb{E}[G^{1-\gamma}]$; a condition which can be reformulated as shown in Equation (A.48):

$$H > \frac{H^{1-\gamma}}{\rho\mathbb{E}[G^{1-\gamma}]} \iff \rho\mathbb{E}[G^{1-\gamma}] > H^{-\gamma} = \rho\mathbb{E}_0\left[G^{-\gamma}\right]\widetilde{R} \iff \widetilde{R} < \bar{G}. \quad \text{(A.48)}$$

This inequality is fulfilled for all three cases mentioned in Equations (17) and (18). Hence, the expected utility from consumption increases when H decreases. From the proof of item 2, we know that H decreases when the interest rate increases, including the case where the debt tax shield applies versus the case where it does not apply. Hence, the claim in item 2 is proven.

Proof of item 3

From Equation (A.11) it holds that

$$\frac{C^a_{t+1}}{C^a_t} = \frac{1-F_{t+1}}{1-F_t} F_t G = \frac{(1-F_{t+1})H}{\frac{1}{F_t}-1} \frac{G}{H} = \frac{G}{H}, \tag{A.49}$$

where the last equality is a simple rewriting of Equation (A.35). Because of the linear sharing rule property, this is also the growth rate of consumption on the individual household level:

$$\frac{C^a_{t+1}}{C^a_t} = \frac{C_{t+1,j}}{C_{t,j}} = \frac{1}{H} G. \tag{A.50}$$

which verifies Equation (23).

Proof of item 4

Equation (24) follows directly from Equation (9) by plugging Equations (3) and (8) in. The expression for I^a_{t-1} follows directly from $I^a_{t-1} = W^a_{t-1} F_{t-1}$ and $W^a_{t-1} = I^a_{t-2} G_{t-1}$.

B Proof of Theorem 2

Proof of item 1

The evolution of wealth from Equation (10) can be rewritten as:

$$\begin{aligned} W_{t,j} = & I^a_{t-1} \left(\left(\alpha_{t-1,j} - \frac{1}{n} \right) (1-\widetilde{\tau}) + \frac{1}{n} \right) G_t + I^a_{t-1} \left(\alpha_{t-1,j} - \frac{1}{n} \right) (\widetilde{\tau} - \tau_E) - \\ & I^a_{t-1} \left(\left(\alpha_{t-1,j} - \frac{1}{n} \right) (1-\tau_E) + \frac{1}{n} \right) \frac{L}{1+L} \widehat{R} + \frac{1}{1+L} I^a_{t-1} \tau_E \left(\alpha_{t-1,j} - \frac{1}{n} \right) + \\ & (\beta_{t-1,j} + \delta_{t-1,j}) \widetilde{R} + \frac{I^a_{t-1}}{n} \frac{L}{1+L} r (\tau_B - \widehat{\tau}_C). \end{aligned} \tag{A.51}$$

Because investors aim at a linear risk-sharing rule, the bond position has to remove the (predictable) terms not related to G_t. Hence:

$$\begin{aligned}\beta_{t-1,j}+\delta_{t-1,j}=&\frac{I^a_{t-1}}{\widetilde{R}}\Bigg[\left(\left(\alpha_{t-1,j}-\frac{1}{n}\right)(1-\tau_E)+\frac{1}{n}\right)\frac{L}{1+L}\widehat{R}-\\&\left(\alpha_{t-1,j}-\frac{1}{n}\right)(\widetilde{\tau}-\tau_E)-\frac{1}{1+L}\tau_E\left(\alpha_{t-1,j}-\frac{1}{n}\right)-\\&\frac{1}{n}\frac{L}{1+L}r\left(\tau_B-\widehat{\tau}_C\right)\Bigg].\end{aligned}\tag{A.52}$$

Collecting and reorganizing terms results in the expression in Equation (26).

Proof of items 2 and 3

With the bond position from Equation (26), Equation (A.51) becomes:

$$W_{t,j}=I^a_{t-1}\left(\left(\alpha_{t-1,j}-\frac{1}{n}\right)(1-\widetilde{\tau})+\frac{1}{n}\right)G_t.\tag{A.53}$$

Household j's consumption is then given by inserting the expressions for the equity and the bond position, $E_{t,j}=\frac{1}{1+L}I^a_t\alpha_{t,j}$, and Equation (26):

$$\begin{aligned}C_{t,j}=&W_{t,j}-E_{t,j}-(\beta_{t,j}+\delta_{t,j})\\=&I^a_{t-1}\left(\left(\alpha_{t-1,j}-\frac{1}{n}\right)(1-\widetilde{\tau})+\frac{1}{n}\right)G_t-\frac{1}{1+L}I^a_t\alpha_{t,j}-\\&\alpha_{t,j}I^a_t\frac{L}{1+L}-\frac{1}{\widetilde{R}}I^a_t\left(\alpha_{t,j}-\frac{1}{n}\right)\left(\frac{L}{1+L}r\left(\tau_B-\widetilde{\tau}\right)-\widetilde{\tau}\right)\end{aligned}\tag{A.54}$$

$$=I^a_{t-1}\left(\left(\alpha_{t-1,j}-\frac{1}{n}\right)(1-\widetilde{\tau})+\frac{1}{n}\right)G_t-\frac{1}{n}I^a_t-I^a_t\left(\alpha_{t,j}-\frac{1}{n}\right)Y\left(1-\widetilde{\tau}\right),\tag{A.55}$$

from which the consumption-wealth ratio follows:

$$\frac{C_{t,j}}{I^a_{t-1}G_t}=\left(\alpha_{t-1,j}-\frac{1}{n}\right)(1-\widetilde{\tau})+\frac{1-F_t}{n}-F_t\left(\alpha_{t,j}-\frac{1}{n}\right)Y(1-\widetilde{\tau}).\tag{A.56}$$

Because of the linear risk sharing rule, every individual household j's consumption share is constant. We denote it by $\omega_j\equiv\frac{C_{t,j}}{I^a_{t-1}G_t(1-F_t)}$. It thus holds that

$$\left(\omega_j-\frac{1}{n}\right)(1-F_t)=\left(\alpha_{t-1,j}-\frac{1}{n}\right)(1-\widetilde{\tau})-\left(\alpha_{t,j}-\frac{1}{n}\right)(1-\widetilde{\tau})\,F_tY.\tag{A.57}$$

For shorter hand notation we define:

$$\widetilde{\alpha}_{t,j} = \alpha_{t,j} - \frac{1}{n} \tag{A.58}$$

and use this to rewrite (A.57) as

$$\left(\omega_j - \frac{1}{n}\right)(1 - F_t) = \widetilde{\alpha}_{t-1,j}(1 - \widetilde{\tau}) - F_t Y \widetilde{\alpha}_{t,j}(1 - \widetilde{\tau}). \tag{A.59}$$

At time $t = N$, it holds that

$$\left(\omega_j - \frac{1}{n}\right) = \widetilde{\alpha}_{N-1,j}(1 - \widetilde{\tau}). \tag{A.60}$$

Working backwards, it holds at time $t = N - 1$ that

$$\left(\omega_j - \frac{1}{n}\right)(1 - F_{N-1}) = \widetilde{\alpha}_{N-2,j}(1 - \widetilde{\tau}) - F_{N-1} Y \widetilde{\alpha}_{N-1,j}(1 - \widetilde{\tau}) \qquad \Leftrightarrow \tag{A.61}$$

$$\widetilde{\alpha}_{N-1,j}\left(1 - F_{N-1}\right) = \widetilde{\alpha}_{N-2,j} - F_{N-1} Y \widetilde{\alpha}_{N-1,j} \qquad \Leftrightarrow \tag{A.62}$$

$$\widetilde{\alpha}_{N-2,j} = \widetilde{\alpha}_{N-1,j}\left(1 - F_{N-1} + F_{N-1} Y\right). \tag{A.63}$$

Similarly, at time $t = N - 2$ it holds that

$$\left(\omega_j - \frac{1}{n}\right)(1 - F_{N-2}) = \widetilde{\alpha}_{N-3,j} - F_{N-2} \widetilde{\alpha}_{N-2} Y \qquad \Leftrightarrow \tag{A.64}$$

$$\widetilde{\alpha}_{N-1,j}\left(1 - F_{N-2}\right) = \widetilde{\alpha}_{N-3,j} - F_{N-2} \widetilde{\alpha}_{N-2} Y. \tag{A.65}$$

Inserting $\widetilde{\alpha}_{N-2}$ from Equation (A.63), we arrive at

$$\widetilde{\alpha}_{N-3,j} = \widetilde{\alpha}_{N-1,j}\left(1 - F_{N-2} + F_{N-2} Y\left(1 - F_{N-1} + F_{N-1} Y\right)\right). \tag{A.66}$$

More generally, $\widetilde{\alpha}_{t,j}$ satisfies the backwards difference equations

$$\widetilde{\alpha}_{t,j}(1 - \widetilde{\tau}) = \left(\omega_j - \frac{1}{n}\right) Z_t, \tag{A.67}$$

in which

$$Z_t = 1 - F_{t+1} + F_{t+1} Y Z_{t+1}. \tag{A.68}$$

With the terminal condition $Z_N = 0$, we have an explicit solution for Z_t in Equation (A.68):

$$Z_t = \frac{H^{N-t} - Y^{N-t}}{H - Y} \frac{(H-1)}{H^{N-t} - 1} = \frac{\sum_{k=0}^{N-t-1} \left(\frac{Y}{H}\right)^k}{\sum_{k=0}^{N-t-1} \left(\frac{1}{H}\right)^k}. \tag{A.69}$$

To find an explicit solution for consumption and investments, it is important to disentangle the relationship between ω_j and $\alpha_{t,j}$. From Equation (A.55):

$$C_{0,j} = W_{0,j} - \frac{1}{n} I_0^a - I_0^a \left(\alpha_{0,j} - \frac{1}{n}\right) Y(1 - \widetilde{\tau}). \tag{A.70}$$

With

$$C_0^a = W_0^a (1 - F_0), \tag{A.71}$$

it then holds that

$$\omega_j = \frac{C_{0,j}}{C_0^a} = \frac{W_{0,j}}{W_0^a (1 - F_0)} - \frac{W_0^a F_0}{n W_0^a (1 - F_0)} - \frac{W_0^a F_0}{W_0^a (1 - F_0)} \left(\alpha_{0,j} - \frac{1}{n}\right) Y(1 - \widetilde{\tau}) \tag{A.72}$$

$$= \frac{W_{0,j}}{W_0^a (1 - F_0)} - \frac{F_0}{1 - F_0} \left(\left(\alpha_{0,j} - \frac{1}{n}\right) Y(1 - \widetilde{\tau}) + \frac{1}{n} \right). \tag{A.73}$$

Plugging (A.67) in, gives

$$\omega_j (1 - F_0) = \frac{W_{0,j}}{W_0^a} - F_0 \left(\left(\omega_j - \frac{1}{n}\right) Z_0 Y + \frac{1}{n} \right) \quad \Leftrightarrow \tag{A.74}$$

$$\left(\omega_j - \frac{1}{n}\right) (1 - F_0 + F_0 Y Z_0) = \frac{W_{0,j}}{W_0^a} - \frac{1}{n} \quad \Leftrightarrow \tag{A.75}$$

$$\omega_j - \frac{1}{n} = \frac{\frac{W_{0,j}}{W_0^a} - \frac{1}{n}}{1 - F_0 + F_0 Y Z_0} = \frac{\frac{W_{0,j}}{W_0^a} - \frac{1}{n}}{Z_{-1}}. \tag{A.76}$$

With $D = 1/Z_{-1}$ and because $\frac{W_{0,j}}{W_0^a} = \alpha_{0-,j}$, this can be written as $\omega_j - \frac{1}{n} = D\left(\alpha_{0-,j} - \frac{1}{n}\right)$.

To complete the proof, we must show that Z_t is a decreasing sequence in $[1, \infty)$. It is easily seen that $Z_{N-1} = 1$. For the remainder, we show that $Y > 1$, because given this, it follows from Equation (32) that Z_t is decreasing. It holds that

$$Z_t = \frac{\sum_{k=0}^{N-t-1} \left(\frac{Y}{H}\right)^k}{\sum_{k=0}^{N-t-1} \left(\frac{1}{H}\right)^k} \quad \text{and} \quad Z_{t-1} = \frac{\sum_{k=0}^{N-t} \left(\frac{Y}{H}\right)^k}{\sum_{k=0}^{N-t} \left(\frac{1}{H}\right)^k}. \tag{A.77}$$

Hence,

$$Z_{t-1} - Z_t = \frac{\sum_{k=0}^{N-t} \left(\frac{Y}{H}\right)^k \sum_{p=0}^{N-t-1} \left(\frac{1}{H}\right)^p - \sum_{k=0}^{N-t-1} \left(\frac{Y}{H}\right)^k \sum_{p=0}^{N-t} \left(\frac{1}{H}\right)^p}{\sum_{k=0}^{N-t} \left(\frac{1}{H}\right)^k \sum_{k=0}^{N-t-1} \left(\frac{1}{H}\right)^k}. \tag{A.78}$$

To prove that this difference is positive, we ignore the denominator, that is trivially positive, and look at the numerator in the following. It holds that

$$\sum_{k=0}^{N-t} \sum_{p=0}^{N-t-1} Y^k H^{-(k+p)} - \sum_{k=0}^{N-t-1} \sum_{p=0}^{N-t} Y^k H^{-(k+p)} = \tag{A.79}$$

$$\sum_{k=0}^{N-t-1} \sum_{p=0}^{N-t} Y^p H^{-(k+p)} - \sum_{k=0}^{N-t-1} \sum_{p=0}^{N-t} Y^k H^{-(k+p)} = \tag{A.80}$$

$$\sum_{k=0}^{N-t-1} \sum_{p=0}^{N-t-1} H^{-(k+p)} \left(Y^p - Y^k\right) + \sum_{k=0}^{N-t-1} H^{-(k+p)} \left(Y^{N-t} - Y^k\right) = \tag{A.81}$$

$$\sum_{k=0}^{N-t-1} H^{-(k+p)} \left(Y^{N-t} - Y^k\right) \quad \text{(by symmetry)}. \tag{A.82}$$

When $Y > 1$, this expression is clearly positive. So it remains to verify that $Y > 1$. When it is optimal to remain unlevered ($L = 0$), this follows immediately from Equation (29). If there is a tax advantage to using debt and the debt tax shield applies we have:

$$Y = \frac{(\widetilde{R} - \widetilde{\tau}) + \frac{L}{1+L} r \left(\tau_B - \widehat{\tau}\right)}{(1-\widetilde{\tau})\widetilde{R}} > 1 \;\Leftrightarrow\; (\widetilde{R} - \widetilde{\tau}) + \frac{L}{1+L} r \left(\tau_B - \widehat{\tau}\right) > (1-\widetilde{\tau})\widetilde{R} \;\Leftrightarrow \tag{A.83}$$

$$r\widetilde{\tau}(1-\tau_B) + \frac{L}{1+L} r(\tau_B - \widehat{\tau}) > 0 \quad\Leftrightarrow\; \widetilde{\tau}(1-\tau_B) + \frac{L}{1+L}(\tau_B - \widehat{\tau}) > 0. \tag{A.84}$$

Upon inserting the optimal degree of leverage we finally get:

$$\widetilde{\tau}(1-\tau_B) + \frac{g_M}{g_M + \frac{1-\widetilde{\tau}}{1-\tau_B}\left(\bar{g} - g_M\right)}(\tau_B - \widetilde{\tau}) > 0 \;\Leftrightarrow \tag{A.85}$$

$$\widetilde{\tau}(1-\widetilde{\tau})(1-\tau_B)(\bar{g} - g_M) + \tau_B(1-\widetilde{\tau})g_M(1-\tau_B) > 0, \tag{A.86}$$

which verifies that $Y > 1$, thereby that $Z_t < Z_{t-1}$ and with $Z_N = 1$ that Z_t is a decreasing sequence in $[1, \infty)$. Hence, poorer households' equity shares increase over time and richer households' decrease. The equity share, $\alpha_{t,j}$, follows immediately from manipulating Equation (A.67):

$$\alpha_{t,j} = \frac{1}{n} + \frac{\left(\omega_j - \frac{1}{n}\right)}{1-\widetilde{\tau}} Z_t, \tag{A.87}$$

where Z_t is given by Equation (32). Plugging Equation (27) into Equation (31) gives

$$\alpha_{0,j} = \frac{1}{n} + \frac{\frac{W_{0,j}}{W_0^a} - \frac{1}{n}}{Z_{-1}\left(1 - \widetilde{\tau}\right)} Z_0, \tag{A.88}$$

which, with $\frac{W_{0,j}}{W_0^a} = \alpha_{0-.j}$ is equal to

$$\alpha_{0,j} = \frac{1}{1} n + \frac{Z_0}{Z_{-1}} \left(\alpha_{0-,j} - \frac{1}{n} \right) \frac{1}{1 - \widetilde{\tau}}. \tag{A.89}$$

For $N \to \infty$, this converges to

$$\alpha_j = \frac{1}{n} + \left(\alpha_{0-,j} - \frac{1}{n} \right) \frac{1}{1 - \widetilde{\tau}}, \tag{A.90}$$

which completes the proof of items 2 and 3.

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