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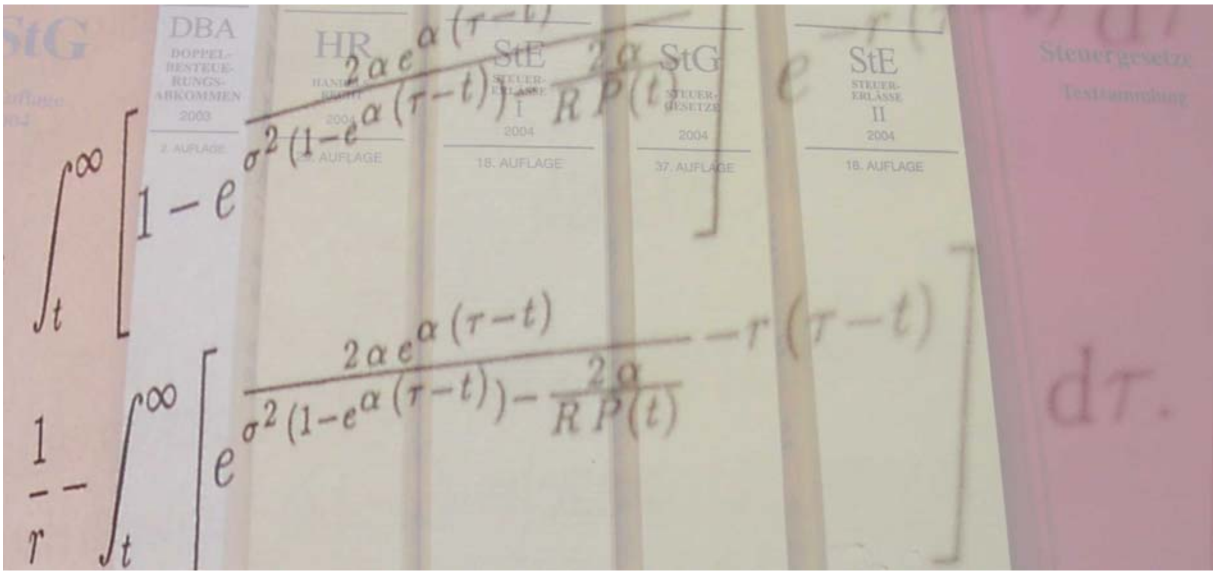

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Group Simulation and Income Tax Statistics - How Big is the Error?

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Abstract: Microsimulation based on income tax statistics may be useful in tax reform discussions. Unfortunately, access to appropriate data is still rather restricted and expensive for ad-hoc analyses, or individual data is often even not available at all. In this paper we take Germany and its data situation as a proxy for many countries' restrictions in terms of tax data availability. Analyzing how much reliability and robustness of results we lose if we employ group simulation instead of microsimulation, we compare both methods. Investigating tax scale effects by the group model leads to very good results. Determining the financial effects of modified tax bases, the deviation from the microsimulation results increases especially if tax base cuts vary between taxpayers. In addition, we take account of the class with taxpayers with a negative taxable income. Neglecting this class we identify a systematic underestimation of the financial consequences of a modified tax base with the group model assuming a progressive tax scale. If the group simulation data is not arranged according to the taxable income but rather according to the total amount of income we find in tendency higher deviations from the microsimulation results. Quantifying tax revenue effects of alternative tax settings the group simulation model represents a good compromise between the desire to capture the complex reality and the achievable accuracy when facing limited resources and data. Our group simulation model will be of major interest especially for analyses of rather old data, as sufficiently detailed data for micro analyses is usually missing.

Keywords: microsimulation, group simulation, tax revenue, personal income tax, tax statistics
JEL Classification: C81, H24

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1 Introduction

Employing microsimulation models on income tax statistics usually aim to analyze fiscal and distributive issues of taxation. These are important fields of research. The results may be useful in tax reform, budget and income distribution discussions and therefore may contribute substantially to solving these three major economic questions. However, access to appropriate data is still rather restricted and expensive for ad-hoc analyses. In addition, in case of analyses based on data from previous assessment periods individual data even often is not available at all.

In the following investigation we take Germany and its data situation as a proxy for many countries' restrictions in tax data availability. This analysis enables us to draw some general conclusions about how to deal with these limitations in future research in countries with a highly developed tax administration and tax statistics but insufficiently detailed and published tax data.

After the amendment of the German Act on Fiscal Statistics in 1996 it was for the first time possible to consolidate the individual data records from the local statistic offices centrally and to use them for auxiliary and special analyses (cf. Zwick, 2001:640, see further Dell, 2006). Now, the data can be prepared more flexibly and used for microsimulations for research and policy purposes.

Because of the generally limited access to micro data or for reasons of economy it is sometimes recommendable for several types of analyses of tax revenue effects to refer instead to classified data from income tax statistics.

This problem has been addressed by several researchers. E.g., Zandvakili (1994) points out that micro data usually is superior to aggregated data with comparable variable definition.

A vast body of literature examines the impact of (progressive) income taxation on income distribution and revenues referring to different sources of data. For a historical overview of US tax policy and income distribution cf. e.g., Brownlee (2000), Atkinson and Bourguignon (2000) with various authors' contributions. For a focus on US and UK income inequality see Lindert (2000). E.g., Berglas (1971) compares the effects of income tax on the distribution of income on the basis of a lognormal distribution of income and further, the share of income tax in national income in different countries while considering effects of tax scales and erosion of the tax base. Kakwani (1977) focuses on the problem of measuring progressivity in taxation and public expenditure and conducts an inter-country comparison using group data from the official income tax statistics. Kraus (1981) employs such data as well to investigate income inequality. Loizides (1988) uses group data from the official Greek tax statistics to measure progressivity effects. Differences between twelve OECD countries are identified by Wagstaff, van Doerslaer, van der Burg et al. (1999) and Wagstaff and van Doorslaer (2001) using household survey and grouped OECD data. Atkinson (1980) analyzes the impact of taxation on horizontal equity on a micro data basis. Cowell (1984) and Zandvakili (1994) examine micro data from household surveys to identify redistributive effects of taxation. Merz (2000) employs micro data from the German income tax statistics, analyzes the redistributional impact of the German tax system, and finds lower inequality for self-employed persons than for employees. For an overview see also Atkinson and Bourguignon (2000) with contributions by several authors.

Based on the seminal work of Orcutt (1957) on microsimulation a new field of research emerged. Orcutt, Merz and Quinke (1986) and Citro and Hanushek (1991) provide contributions of various authors and describe the opportunities and limitations of research based on microsimulation models for policy support purposes. Several applications of such models in different countries are presented and methodological aspects, data requirements, computing technology etc. are discussed. Pudney and Sutherland (1994) discuss the reliability of microsimulation results are. Further research applying microsimulation tax-benefit models provide a deep insight into the tax effects based on micro data of several countries. Sutherland (1995) gives an overview over static microsimulation models in five European countries and prepares the field for a European model. Hancock (1997), Sutherland (1997, 2001), Immervoll, O'Donoghue and Sutherland (1999) and Immervoll and O'Donoghue (2001b) illustrate the types of analysis that can be performed with specific microsimulation models, and discuss an appreciation of the constraints and assumptions that shape the analyses. Callan and Sutherland (1997) explore the prospects and limitations of such models referring to a case study, whereas Galler (1997) compares a continuous-time approach to dynamic microsimulation and discrete-time models. An analysis of the distributional effect of replacing turnover tax with a value-added tax applying a Finnish microsimulation model is offered by Salomaki (1996). Tax-benefit models are presented in Callan, O'Donoghue and O'Neill (1996), Bourguignon, O'Donoghue, Sastre-Descals et al. (1997), Mitton (1998), Redmond, Sutherland and Wilson (1998), Immervoll and O'Donoghue (2001a), Immervoll (2004), Verbist (2004, 2005) and Spadaro (2005) investigating tax changes. Atkinson (2002) stresses that in specific cases, tax data may be superior to data from household surveys employing UK tax data. An overview of international tax microsimulations models is given by O'Hare and Gupta (2000). Spahn (1975) applies a group simulation model to synthetic German data. Gyárfás and Quinke (1990), Bork and Petersen (2000), Wagenhals (2001) and Haan and Steiner (2005) employ microsimulation to analyze German tax reform effects. Wagenhals (2004) and Peichl (2005) describe the recent literature on microsimulation models relying on German data.

Piketty (2003) highlights French tax data deficits and estimate income inequality in France on the basis of tax statistics. Piketty and Saez (2003, 2006) and Saez and Veall (2004) look at US and

Canadian grouped tax data. Dell (2006) uses group data from the German tax return statistics, identifies several breaks in data over time, and stresses certain limits of recent data on tax bases and taxes paid. All of them investigate the tax impact on distribution, especially on top incomes over the twentieth century. Atkinson and Leigh (2004) compare the UK results with other countries' estimates. Morrisson (2000) offers a survey of various related studies.

Whereas several papers point out that using group data limits the reliability of their studies in general (cf. Kakwani, 1977:75, Orcutt, 1982, Caldwell, 1985, McClung, 1986, Wagstaff and van Doorslaer, 2001:313), there is no analysis about the extent of inaccuracy arising from data deficiencies.

In order to find out how much reliability and robustness of results we lose if we employ group simulation instead of microsimulation for the highlighted reasons, we compare both methods. We apply microsimulation to micro data and group simulation to classified data from the same data basis. The results allow us to draw conclusions about the opportunities and limitations of group simulation instead of microsimulation models.

The remainder of this paper begins with an introduction to the tax statistics of the German Federal Statistical Office in section 2. In section 3 we describe the main characteristics, advantages and limitations of micro and group simulation models. We present our model in section 4 and the simulation results in section 5. On this basis we summarize and draw final conclusions on the applicability, reliability and robustness of results obtained from the alternative methods in section 6.

2 Tax Statistics of the Federal Statistical Office

Income statistics are secondary statistics, i.e. the tax authorities allocate data to the tax statistics that is collected during the tax assessment procedure. They hence consist of authentic data. The data is not collected through questionnaires but extracted from personal tax assessments recorded by the fiscal administration for statistical reasons. The income tax statistics, however, are only assembled all three years by the German Federal Statistical Office with a time-lag of at least of four or five years.

A multitude of data from wage tax cards, tax returns and from official tax assessment notes are documented in the tax statistics. Married couples that are jointly assessed are regarded legally as one tax payer (cf. for problems on referring to tax unit as individual or couple e.g., Wagstaff and van Doorslaer, 2001:307). The 1995 tax statistics contain approximately 30 million data sets of 38 million persons with around 400 attributes per data set (cf. Zwick, 2001:641). Besides technical and socioeconomic information these data sets include data that is necessary to determine the individual tax base and people's personal tax liabilities. In line with the official calculation procedure for tax assessment all corrections, e.g. for special expenses and expenses for extraordinary financial burdens, are considered as appropriate. Beginning with the “income from different sources of taxable income” these adjustments are conducted and finally produce the tax base, i.e. the “taxable income“. In addition to the tax base the tax liability is documented in the income tax statistics. Applying the tax scale to the taxable income leads to the “tax scale income tax”. Then, tax credits, tax pre-payments (e.g., by wage tax and source taxes), tax refunds etc. have to be taken into account to arrive at the assessed tax liability.

The German Federal Statistical Office publishes part of these data in tables. These tables distinguish between classes of "total income" and classes of "taxable income". The total amount of income is a kind of preliminary tax base, i.e. a tax base before special individual expenses and expenses for extraordinary financial burdens. Thus, the tables contain the aggregated value of the underlying attributes from the individual data sets of the taxpayers or certain groups of taxpayers, e.g. subject to the basic or splitting tax scale. The published tables provide group-specific information about the tax base and the assessed tax. The tables used for group simulation only provide mean values for each attribute and class. As a result of aggregating data in each tax class overall a substantial information loss arises in comparison to the corresponding, non-disclosed individual micro data sets.

3 Micro vs. Group Simulation

Referring to the most important distinctive feature - the degree of aggregation of the applied data - in economics and the social sciences we find three basic types of simulation model:

- Models that are essentially based on the aggregates from the national accounting system, like macroeconomic models and general equilibrium models (high aggregation level),
- Group models that refer to selected attributes of homogeneous groups of economic units (medium aggregation level), and
- Microanalytic models that focus on individual micro units (strong disaggregation).

Macroeconomic models and equilibrium models are not generally suitable for analyzing income tax revenue as, due to the high degree of aggregation, attributes of the taxpayers and structural factors are insufficiently considered both in the model and in the results.

In contrast, the more intensively disaggregated group models and the microanalytic models offer structural advantages. Generally, group models have a relatively simple and transparent structure compared with the microanalytic models. This facilitates their implementation and modification and makes them a flexible and low cost instrument for investigating revenue effects. This advantage has to be offset against the mentioned information loss caused by using data aggregated with respect to a specific attribute. If micro data is not available, any analysis of different combinations of characteristics will be fairly limited. Hence the field of application of group models is restricted by the underlying aggregation pattern. Often, however, these restrictions are acceptable in tax revenue analyses. A high degree of disaggregation that can only be achieved by microanalytic simulation models is particularly necessary for analyzing distributive effects and behavioral simulations as well as for comprehensive simulations of various tax and transfer systems.

Microeconomic models take explicit account also of taxpayers' individual attributes and hence allow us to determine the tax base and tax liability more precisely. It is therefore possible to make a more accurate and differentiated assessment of the revenue effects of e.g. a tax reform.

In a microanalytic simulation each individual micro unit with its attributes is referred to directly. This can be realised on the basis of individual cases, a sample or the parent population. The advantage of comprehensive and detailed structural information can only be exploited if an appropriate multiplicity of attributes of the micro units is available in the database. In order to achieve a simulation as close to reality as possible interdependencies of tax reform and individual behaviour have to be taken into account. Thus, we have to refer to the relevant elasticities, utility functions etc. in the model on either an empirical or theoretical basis. This increases the complexity of the model as well as the number of attributes.

Even if the microanalytic models are theoretically superior to the group models, the required specification and format of the data and the necessity to update it often limit or even prevent the application of microsimulations. In particular for ad-hoc analyses or analyses of earlier tax periods we have to fall back to the published aggregated data as no other detailed data is available. In these cases only group simulation models can be employed.

However, many group simulations on tax revenue tend to lead to too small values for the tax revenues. This is because progressive income taxation is usually not simulated correctly. Referring to aggregate income per income class and aggregate income tax per class instead of exact individual income, the effects of the progressive income tax function cannot be simulated adequately (cf. in addition Gyárfás, 1990:19 and 32-34, who determines the upper limit of the systematic underestimation). This inadequacy can be compensated by applying specific distribution functions. Group simulation therefore becomes an attractive and powerful instrument and alternative to microsimulation models.

The formal reproduction of an empirical frequency distribution from aggregated data can be achieved in principle by two methodological approaches:

- Applying analytic distribution functions whose parameters are derived from empirical material by approximation, or
- Applying interpolation functions.

The mathematical approximation of the analytic distribution functions to empirical distributions is very time-consuming and complex. Furthermore, there are often substantial deviations in particular in the upper and lower income classes. If no acceptable mathematical approximation can be achieved we have to abstain from a theoretical approach to empirical income distribution and rather conduct an interpolation. Nevertheless, it has to be noted that the advantage of using an analytic distribution function is limited as a useful economic interpretation of the assumed parameters is usually not given.

Tax revenue analyses can normally be conducted without an analytic income distribution.[1] We derive the results presented in the following by applying a group simulation model and determining the distribution of income by means of a linear interpolation of the group simulation. However, no continuous function but rather an arithmetical series, i.e. a discrete function, is chosen for the distribution of income. Generating discrete income distribution functions is appropriate for the tax revenue analysis since the domain of the income tax scale function contains only natural numbers and thus discrete arguments.[2] In addition, the aggregate taxable income of all taxpayers in each tax class is considered in the interpolation (cf. further Gyárfás 1990:30–31).

The presented discrete model for simulating the personal income taxation based on classified data ensures that in each class aggregated taxable incomes of each class and the frequency distribution of the number of taxpayers per class are both identical to the original sums of the micro data per class. Therefore, a degree of precision in disaggregation can be achieved that leads in each class to a 100% adjustment of aggregated taxable incomes of each class, as indicated in the tax statistics.

1 As analytical theory-based income distributions only approximate real world distribution, e.g. a log-normal income distribution (e.g., Berglas, 1971:534) or a Pareto distribution (e.g., Piketty and Saez, 2003:6, Saez and Veall, 2004:5), it is preferable to deduce the income distribution from the available data.

2 In accordance with § 32 para. 2 EStG, the income tax scale only has to be applied to full DM (deutschmark) or euro amounts.

4 The Model

In the following, we introduce a discrete income tax simulation model based on classified data from German Fiscal Statistics.[3] The aim of this group model is to identify the revenue effects of alternative tax rules or systems, particularly the fiscal consequences of specific tax regulations, rapidly and flexibly. The model is supposed to refer to available aggregate data from the income tax statistics. After presenting the group model we compare the results of micro and group simulation calculations in order to assess the accuracy of the group model.

4.1 Discrete income distribution

The absolute frequency of taxpayers with a specific taxable income *TI* is $h_{(TI)}$ and yields from the closed income interval *i* with the interval bounds $[a_i,b_i]$, with $a_{i+1}=b_{i+1}$, of the discrete density function of the taxpayers:

(1.1) $$h_i = \sum_{TI=a_i}^{b_i} h_{(TI)}.$$

This set of numbers is a unique transformation of a set of natural numbers (taxable income) on a set of integers (absolute frequency of the taxpayers).

The sum of the taxable income of the taxpayers in the interval *i* is TI_i and can be determined as follows from the density function:

(1.2) $$TI_i = \sum_{TI=a_i}^{b_i} h_{(TI)} TI.$$

Applying the income tax scale to the tax base *TI*, neglecting preliminary special tax scale regulations,[4] we receive income tax $t_{(TI)}$. The sum of the determined income tax of all taxpayers of the interval *i* is T_i and is given by:

(1.3) $$T_i = \sum_{TI=a_i}^{b_i} h_{(TI)} t_{(TI)}.$$

3 For a continuous-time approach cf. e.g. Galler (1997).

4 E.g. a differing income tax may result from applying the “exemption with progression” rule or specific tax rates for extraordinary earnings.

4.2 Taxable income class

As already described, the published tables from the income tax statistics – separated into taxpayers underlying the basic scale and taxpayers underlying the splitting tax scale – include aggregate data for a variety of tax relevant facts. For each class the number of taxpayers and further facts are displayed in deutschmark (DM). It is appropriate to conduct simulations for tax revenue analysis purposes with a group model using data grouped with respect to classes of taxable income, since the range of values for the tax base of the taxpayers in each class is explicitly given and thus, the interpolation of the distribution of the taxpayers is limited to this interval.

We use information relating to the number of taxpayers with a taxable income, the sum of the taxable income of these taxpayers and the sum of assessed income tax from the income tax statistics. This database can formally be described for taxpayers subject to the basic or splitting tax scale as follows:

Given are classes of "taxable income" TI for i=1 to n classes with the class limits $[a_i, b_i]$, where $a_1=-\infty$, $b_1=0$, $a_2=1$ and $b_n=\infty$. For every class i we know:

- the class frequency h_i (number of taxpayers of the class i for whom a taxable income has been assessed),
- the sum of the taxable income TI_i of the taxpayers of class i, and
- the sum of the assessed income tax AT_i of the taxpayers of the class i.

The assessed income tax AT_i of all taxpayers results from the application of all relevant tax rate regulations, tax reductions and tax base additions without imputable taxes .

Unfortunately, the income tax statistics does not include the “tax scale income tax” but the sum of the “assessed income tax” of each class. In contrast to the “assessed income tax” the “tax scale income tax” results from the assessment process at a stage before special regulations, tax reductions and tax base additions are considered. Furthermore, the absolute frequency of the taxpayers with a specific taxable income $h_{(TI)}$, the sum of these taxable income $h_{(TI)}\ TI$ as well as the corresponding income tax from $h_{(TI)}\ t_{(TI)}$ cannot be found in the aggregate data of the income tax statistics. Only the average taxable income of each class $\overline{TI}_i = \frac{TI_i}{h_i}$ can be determined by dividing the sum of the taxable incomes and the number of taxpayers of the class. Further information that may be helpful to analyze the distribution of the taxpayers within the class is not available. Since the total assessed tax T is the result of assessment after considering all individual relevant tax regulations no additional information about the distribution of the taxpayers can be gained by referring to sums of assessed income tax in the respective income classes (AT_i) published in the income tax statistics.[5] Even if we assume identical tax bases for every taxpayer of an income class different income tax assessments may arise, as specific tax regulations may lead to different reductions and additions. A strict functional relation between the assessed income tax and the assessed tax base "taxable income" cannot be assumed.

Applying this aggregated data and a discrete group simulation model we determine the financial consequences of alternative scenarios.

We hence have to be aware of the fact that during a simulation based on classified data the above mentioned problem for progressive income tax scale will occur. If we determine the income tax revenues referring to average taxable income per income class by multiplying the income tax on the average taxable income $t_{(\overline{TI}_i)}$ of the class with the number of taxpayers of the class h_i, the deduced tax revenue will generally be too low. This is due to the fact that within the segment of the progressive rise of the income tax rate, the income tax on the average assessed tax base may not map the effect of the progressive structure precisely. Furthermore, the effects of a transition between two tax scale zones of the tax schedule cannot be reproduced within a class because the average taxable income of the class can lie only in one zone.[6] This affects particularly the simulation of the revenues from reformed tax bases and reformed tax schedules with different tax scale zones.

In the following, in order to reduce these inaccuracies when determining income tax revenues by means of a group simulation based on classified data we develop a discrete model for the taxpayer distribution within a class by applying linear interpolation (cf. Wagstaff and van Doorslaer, 2001:307, Atkinson, 2002:12-15, Saez and Veall, 2004:5). The linear interpolation requires the description of m elements between two numbers z_1 and z_2 with the difference $z_2 - z_1 = d$ in such a way that a finite arithmetic series of numbers emerges whose first element is z_1 and whose $(m+2)$th element is z_2. If $\bar{d}$ denotes the difference of the wanted arithmetical series of numbers, then $z_2 = z_1 + (m+1)\bar{d} = z_1 + d$, i.e. $\bar{d} = \frac{d}{(m+1)}$.

In a first step we assume that the taxpayers in the closed interval i (class) with the interval bounds $[a_i, b_i]$ are equally distributed (cf. also Gyárfás and Quinke, 1993:150). In this case the average

5 This is also valid for the attribute “tax scale income tax” expelled in the income tax statistics since the tax scale induced income tax is influenced by special rate prescriptions as well.

6 Here, in particular, the transition from the zero-zone of the tax schedule that is determined by the basic tax-exempt amount to the next zone is problematic, since in the case of an average taxable income of the class lying below the basic tax-exempt amount the aggregated income tax of the class would be zero.

taxable income of a class is identical to the mid-point of class $\overline{TI}_i = \frac{(a_i + b_i)}{2}$. The sum of the taxable income of all taxpayers of a class is given according to eq. (1.2) by the product of the average taxable income and the number of taxpayers of this class: $TI_i = h_i \overline{TI}_i$. The aggregated income tax of the class can easily be determined by eq. (1.3) since the absolute frequency of the taxpayers for every taxable income within the interval is identical and can be described by $h_{(a_i)} = h_{(a_{i+1})} = ... = h_{(b_i \; 1)} = h_{(b_i)} = \frac{h_i}{(b_i - a_i + 1)}$.

However, the average taxable income of a class is usually not equal to the mid-point of a class. Then, obviously, there is no uniform distribution of the taxpayers in this class. Therefore, an assumption about the distribution of the taxpayers within the class is necessary.

Starting with the uniform distribution a discrete function (arithmetical sequence of numbers) that is strictly monotonously increasing or falling has to be assumed for the distribution of the taxpayers in the class. This function is conditioned on the position of the average taxable income in the class in relation to the mid-point of the class. We presume that the number of taxpayers in the mid-point of the class is equal to the quotient of the total number of the taxpayers of this class and the class breadth, i.e. $h_{\left(\frac{a_i+b_i}{2}\right)} = \frac{h_i}{(b_i - a_i + 1)}$. Then, the problem is reduced to redistributing a certain number of taxpayers between the lower and upper class halves so that the sum of the income of the class corresponds to the empirical value. This redistribution is standardized such that the number of taxpayers at the beginning and end of the class differ exactly by two taxpayers, i.e. $\left| h_{(a_i)} - h_{(b_i)} \right| = 2$. Thus, the difference between the number of taxpayers in the mid-point of the class and the number of taxpayers at the class beginning or the class end is exactly: $\left| h_{(a_i)} - h_{\left(\frac{a_i+b_i}{2}\right)} \right| = \left| h_{(b_i)} - h_{\left(\frac{a_i+b_i}{2}\right)} \right| = 1$, hence one taxpayer.[7] Within the class the number of taxpayers rises and falls with $\frac{2}{(b_i - a_i)}$ whenever the underlying taxable income *TI* is amended by one DM.[8] This standardization ensures

[7] The sign of the difference of the number of taxpayers at the class border and the one in the mid-point of the class it determined by the position of the average taxable income of the class $\overline{TI}_i$ in relation to the mid-point of the class $\frac{a_i + b_i}{2}$. If $\overline{TI}_i > \frac{a_i + b_i}{2}$, then $h_{(b_i)} - h_{\frac{a_i+b_i}{2}} = 1$ and $h_{(a_i)} - h_{\frac{a_i+b_i}{2}} = \; 1$. Whereas if $\overline{TI}_i < \frac{(a_i + b_i)}{2}$ the differences are given by $h_{(a_i)} - h_{\left(\frac{a_i+b_i}{2}\right)} = 1$ and $h_{(b_i)} - h_{\left(\frac{a_i+b_i}{2}\right)} = -1$.

[8] This simplifying procedure is responsible for the number of taxpayers with a specific income $h_{(TI)}$ is not necessarily integer in the model.

the required strict monotony. The degree of redistribution within a class $u_{i,}$ can now be determined by referring to the empirical taxable income of the class:

(1.4)
$$u_i = \frac{TI_i - \frac{a_i + b_i}{2} h_i}{\sum_{TI=a_i}^{b_i} \frac{TI - a_i - \frac{b_i - a_i}{2}}{\frac{b_i - a_i}{2}} TI}.$$

The number of taxpayers with a specific taxable income under the given set of assumptions is:

(1.5)
$$h_{(TI)} = \frac{h_i}{b_i - a_i + 1} + \frac{TI - a_i - \frac{b_i - a_i}{2}}{\frac{b_i - a_i}{2}} u_i.$$

Considering u_i and h_i the number of taxpayers $h_{(TI)}$ and thereby $h_{(TI)}\, TI$ and $h_{(TI)}\; t_{(TI)}$ can be estimated for every taxable income. Inserting the frequencies of the taxpayers from eqn. (1.5) into the eqns. (1.2) and (1.3) we find for every class i that the sum of taxable income TI_i equals exactly the empirical value from the income tax statistics. This is true since $h_{(TI)}$ is determined via TI_i . Furthermore, the total income tax T_i of this class can be estimated.

Proceeding like this when determining the aggregate income tax of a class we succeed in reducing the systematic underestimation in group models fundamentally. If the aggregate tax of a class is determined by multiplying the income tax on the average taxable income of the class with the number of taxpayers of the class under a progressive tax, we receive the minimum level of the possible total tax of the class. If we instead employ a strictly monotonous discrete function that is defined on the basis of the empirically determined number of taxpayers and the sum of the taxable income of the class, then the total tax of a class varies between the theoretical minimum and maximum possible total tax of this class.

5 Comparing Tax Revenues Effects of Microsimulation and Group Simulation Models

5.1 Tax scale simulation based on taxable income

This type of group simulation allows us to obtain quite exact results involving relatively low effort, particularly when simulating different tax scales. The quality of this simulation approach can be emphasized in the following by comparing the results of the microsimulation, carried out by the German Statistical Office, with those of the discrete group simulation model introduced here. The simulations of the German Statistical Office consulted for comparison purposes are carried out on the base of individual data sets from a 10% sample of the 1995 income tax statistics. The 10% sample is a formally anonymizised sample taken from the entirety of the recorded income tax assessments of the 1995 assessment period in the income tax statistics. This sample is a stratified random sample provided by the German Statistical Office (on how to assemble the sample see Zwick, 1998a:261-264, and Zwick, 1998b: 570-573)..

In the following, the simulation of tax patterns is stylized, i.e. aligned with the main characteristics of the tax code. Thus, specific regulations, such as German tax relief for commercial earnings applicable only in 1995, have been neglected. The initial values of the sample and the results of the sample from the simulation were extrapolated to the parent population by the German Statistical Office. On basis of the aggregated data of the extrapolated initial values for the number of taxpayers and the aggregated taxable incomes of the classes, we run simulations using the discrete group model. Since the German Statistical Office defines the lowest income class as having no lower and the upper as having no upper limit, these class borders for group simulation purposes are heuristically determined. Therefore, assuming a uniform distribution, the average taxable income of the class is equated with the mid-point of the class $\overline{TI_i} = \frac{a_i + b_i}{2}$. Thus, the upper limit of the interval is equivalent to twice the mid-point of the class, i.e. $\overline{TI_i} \times 2 = b_i$. This also applies to the lower class limit of the first class, i.e. $\overline{TI_i} \times 2 = a_i$, because this class contains all taxpayers with a taxable income of less than one DM and therefore, the taxable income may even be negative in this class.

The results presented in table 1 show that the differences between the results from using our group simulation model and the results from the simulation by the German Statistical Office based on micro data, both applying the basic tax rate and the 1990 and 1996 income tax scales, are very small. This result is robust even if we analyze the splitting tax scale instead. The observable deviations, as expected, are much lower than the theoretically derived relative underestimation of the tax liability if we refer to the mid-point of the class (cf. Gyárfás, 1990:43, table 1). It is remarkable that the high quality of the group simulation results arises when comparing not only the total tax revenues but also in almost every single class. The sometimes substantial deviations found by other models in the lower and upper income classes (cf. e.g., Piketty and Saez, 2003:55, concerning the heterogeneity in the top income decile) are small if we employ our discrete group simulation model. Moreover, the quality of the results of the discrete group simulation model is not dependent on the class limits chosen by the German Statistical Office. Even for simulations with tax scales whose basic tax-exempt amount does not correspond to the class limits set by the German

Statistical Office, differences of similar structure and dimension occur, i.e. again very small deviations.

Table 1: Tax scale based micro and group simulation of tax revenue for the basic tax scale (1995 income distribution)

no.	taxable income *from ... to under ... DM*		1990 tax scale			1996 tax scale
			income tax in (DM ' 000)		*relative difference*	*relative difference*
			microsimulation (German Statistical Office)	*group simulation*		
1		under 1	-	-	0.0000%	0.0000%
2	1 -	5,670	-	-	0.0000%	0.0000%
3	5,670 -	8,154	180,739	180,734	-0.0028%	0.0000%
4	8,154 -	12,096	965,929	965,970	0.0042%	0.0000%
5	12,096 -	12,366	83,707	83,706	-0.0012%	-0.0685%
6	12,366 -	13,068	219,226	219,228	0.0009%	0.0083%
7	13,068 -	18,036	1,837,818	1,837,820	0.0001%	0.0022%
8	18,036 -	25,002	4,018,826	4,018,823	-0.0001%	0.0008%
9	25,002 -	30,023	4,408,774	4,408,747	-0.0006%	-0.0003%
10	30,023 -	40,013	14,380,662	14,381,434	0.0054%	0.0034%
11	40,013 -	50,004	15,660,133	15,660,329	0.0013%	0.0010%
12	50,004 -	55,728	7,173,713	7,173,689	-0.0003%	-0.0003%
13	55,728 -	58,644	3,108,599	3,108,590	-0.0003%	-0.0003%
14	58,644 -	60,048	1,403,312	1,403,312	0.0000%	0.0001%
15	60,048 -	66,366	5,465,481	5,465,480	0.0000%	-0.0005%
16	66,366 -	70,038	2,652,440	2,652,457	0.0006%	0.0006%
17	70,038 -	75,006	3,013,301	3,013,302	0.0000%	0.0001%
18	75,006 -	100,008	8,784,647	8,782,625	-0.0230%	-0.0230%
19	100,008 -	120,042	3,299,181	3,298,992	-0.0057%	-0.0058%
20	120,042 -	240,084	6,458,174	6,458,172	0.0000%	0.0000%
21	240,084 -	480,168	3,488,455	3,488,455	0.0000%	0.0003%
22	480,168 -	1,000,026	2,549,395	2,549,390	-0.0002%	0.0000%
23	1,000,026 -	and more	7,226,559	7,226,536	-0.0003%	-0.0003%
		total	**96,379,068**	**96,377,792**	**-0.0013%**	**-0.0018%**

Source: German Statistical Office, Wiesbaden; own calculations.

5.2 Tax base deductions simulation based on taxable income

It is desirable to find out whether the degree of precision of our group model obtained for tax scale simulations (5.1) is achievable for the simulation of tax revenue effects caused by reforms of fixed amount tax base deductions as well (e.g. flat amounts).[9] In the following, we focus on the problem of tax deductions from the tax assessment base (cf. O'Donoghue and Sutherland, 1999:576-577). In order to measure the fiscal impact of these deductions, their tax revenue effects are determined by considering a corresponding increase in the tax base within the simulation. Our conclusions can in

9 Since for such facts no microsimulation was carried out by the German Statistical Office no comparison with the group simulation on the basis of an existing concrete example can be presented.

principle be transferred to tax regulations that lead to an increase of the tax base and their tax revenue effects by simulating an adequate tax base reduction.

However, in case of such simulations the differences between micro and group analyses may increase if the underlying fixed amount is not deductible by all taxpayers and further, if the (relative) distribution of the taxable income of the taxpayers who enjoy this deduction does not correspond to the (relative) distribution of the taxable income of all taxpayers. In order to improve the quality of the results of our group model, information about the distribution of the taxpayers enjoying this fixed tax privilege, as far as this information is available, should be considered explicitly in the simulation. From the published income tax statistics, as outlined already, the number of taxpayers and the total amount of fixed amount tax base deductions in thousands of deutschmarks per class is given. We therefore we have information about the distribution among different income classes, but not about the distribution of these amounts among the taxpayers within the classes. If the tables in the income tax statistics do not provide data on the taxable income of the taxpayers who enjoy, then for group simulation purposes we have to fall back on the sum of the taxable incomes of all taxpayers and hence, on the distribution of all taxpayers in this class derived from the group simulation. This may involve a larger deviation from the results of a microsimulation.

Using the symbols defined under 4.1 the problem can be presented formally as follows: From the aggregated data of the income tax statistics we know for each class i the frequency g_i of the existing tax facts (number of taxpayers, who are affected by this fact) and the sum of its value G_i, where the average value of a class is given by $\overline{G_i} = \frac{G_i}{g_i}$. In the case of a fixed tax base deduction G_i is constant for each class. The financial consequences of this tax rule per class arise from the difference ΔT_i between the respective sum of the income tax of the class both including the effects of the deduction (T_i^g) and excluding its effect (T_i):

$$\Delta T_i = T_i^g - T_i ,$$

where

$$T_i^g = h_{(TI)}^g \; t_{(TI+\overline{G_i})} .$$

Here $h_{(TI)}^g$ is the number of taxpayers with a specific TI who are affected by g_i. Furthermore, $t_{(TI+\overline{G_i})}$ denotes the income tax for the tax base TI which is increased by $\overline{G_i}$. The degree of precision of the simulation is also influenced by whether or not we are informed about the sum of the taxable incomes of the taxpayers for the class i who deducted an amount (TI_i^g) due to special fixed tax regulations. Determining u_i and $h^g_{(TI)}$ using the eqns. (1.4) and (1.5) it is relevant if we refer to the taxable income of all taxpayers (TI_i) or to the taxable income (TI^g_i) of those taxpayers who enjoy tax privileges and thus are included in g_i. If TI^g_i is known, then $h^g_i = g_i$. Otherwise u_i has to be determined on basis of TI_i and, for reasons of simplicity, we set $h^g_{(TI)} = h_{(TI)} \frac{g_i}{h_i}$. Proceeding like this, an identical distribution of the taxpayers with a specific taxable income for the respective class is assumed for all examined tax facts.

Precision is further reduced when applying a discrete group simulation model to determine tax revenue effects caused by tax base deductions that vary between taxpayers. This is imaginable in case of e.g. depreciation or loss offset allowances.

Since the actual distribution of the taxpayers cannot be determined from the aggregated data we need appropriate assumptions on the distribution of the underlying tax deductions in each class in analogy to the distribution of the taxable income. This assumption is necessary even if the distribution of the taxpayers and the deductible amount in the income classes and further the sum of the taxable incomes of the taxpayers in question can be taken from the tables of the German income tax statistics. For our analysis, again for reasons of simplicity, we assume a uniform distribution so that for every taxpayer of a class the average value $\overline{G_i} = \frac{G_i}{g_i}$ that can be deduced from the sum of tax deductions of each class is taken as a proxy for the individual amount. Then, in contrast to a fixed amount a different average amount $\overline{G_i}$ may occur for each class.

The results of the microsimulation by the German Statistical Office on income tax revenue effects in case of limited loss offset are compared with those of our discrete group model in table 2.[10] Loss offset restriction is analyzed as it was not possible to compensate losses vertically with positive earnings.

In line with the comparison in table 1 we apply the basic tax scale to determine the income tax. We assume the 1990 tax scale.[11] The relative divergence of the income tax calculated on the basis of the group simulation and the income tax calculated on the basis of the microsimulation is presented in table 2 for each income class as well as for all taxpayers. Furthermore, we distinguish between the basic and the splitting tax scale. The relative difference between the financial consequences of a refusal of the vertical loss offset is shown at the end of the table for both simulation models. The financial consequences arise from the difference of each determined sum of income tax of all

[10] The numbering of the classes in table 1 corresponds to those in the table 1.

[11] Applying the 1990 tax scales in contrast to the 1996 tax scale in the previous section clarifies that the high accuracy remains for various tax scales independent of the class borders chosen by the German Statistical Office.

taxpayers with negative earnings in case of either a complete or limited loss offset (cf. Wagstaff and van Doorslaer, 2001:307).

In addition, the group simulation is carried out on the basis of differently aggregated data. In one case tabulated data from the sample projected by the German Statistical Office is used. This sample only contains data of the taxpayers with a negative income, i.e. the sum of the taxable income of these taxpayers per class (TI_i^g) is known. This group specific information cannot be found in the publicly available statistics. Rather, it was prepared by the German Statistical Office as a special statistical evaluation for this research project only. In contrast, for the group simulation we use the sum of the taxable income of all taxpayers of the class (TI_i) provided in the tabulated data to simulate the distribution of the tax bases within the class. The relevant details for all taxpayers are included in the published statistics.

A comparison of the results of tables 1 and 2 shows that the deviations of the results of the group simulation from those of the microsimulation concentrating on tax revenue effects from tax base deductions (which may be different for every taxpayer) are substantially greater than when simulating different tax scales. In case of an interpolation of the distribution using the class sum of the income of the taxpayers with a negative income (TI_i^g) we find for the aggregated income tax of all taxpayers with negative earnings determined by group simulation relative deviations from the results under microsimulation of -1.9% (basic tax scale) and -3.7% (splitting tax scale). We receive 2.6% (basic tax scale) and 3.0% (splitting tax scale) referring to the sum of the taxable income of all taxpayers of the class (TI_i).

The differences are largest in the lower income classes and decrease as the tax base increases. The greatest relative difference is observed for class 1, which includes taxpayers with a taxable income less than one DM. Since this class is not further subdivided in the income tax statistics but covers a wide range of negative taxable incomes, here the group simulation model is highly inaccurate. As a consequence, estimating the number of taxpayers with positive income greater than the basic tax-exempt amount due to vertical loss offset restriction is rather unreliable. Besides, the results in this class depend on the lower class boundary which must be determined heuristically. Including the class of the taxpayers with a taxable income less than one DM is reasonable only for microsimulation of tax revenue effects if we want to analyze an increase in the tax base - as far as these taxpayers are affected by it.[12] Due to the lack of data, in this case a group model can only arbitrarily lead to similar results as a microsimulation. If the class of taxpayers with a taxable income less than one DM is neglected in simulation, comparing micro and group models leads to

[12] Several studies solely consider taxpayers or households with positive income. Cf. e.g. Zandvakili (1994:479).

relative deviations in tax revenues for all taxpayers with a negative income employing interpolation using TI_i^g of 1.9% (basic tax scale) and -1.8% (splitting tax scale) and further, using TI_i of -3.5% (basic tax scale) and -2.1% (splitting tax scale).

Table 2: Tax base based micro and group simulation of tax revenue and the financial effects using *TI* tables in case of vertical loss offset restriction (1995 income distribution)

TI class No.	relative difference between the results of microsimulation (German Statistical Office) and the discrete group model			
	interpolation by TI_i^g		interpolation by TI_i	
	basic tax scale	splitting tax scale	basic tax scale	splitting tax scale
1	-1.4182%	-100.0000%	194.1274%	263.1967%
2	-34.5701%	-35.6779%	-37.4919%	-38.2353%
3	-20.3230%	-17.8262%	-19.9858%	-17.7484%
4	-14.7191%	-11.8263%	-15.0016%	-11.6785%
5	-9.4212%	-7.9040%	-9.4722%	-7.8956%
6	-11.7048%	-7.3189%	-11.7094%	-7.3079%
7	-9.8775%	-6.5023%	-10.5133%	-6.4304%
8	-6.3505%	-4.3696%	-6.3986%	-4.9910%
9	-5.1233%	-2.9831%	-5.2855%	-3.0239%
10	-3.6476%	-2.7450%	-3.6717%	-3.1454%
11	-3.3690%	-2.6741%	-3.8018%	-3.1458%
12	-2.4027%	-2.4326%	-2.7062%	-2.4803%
13	-2.7901%	-2.3300%	-2.8571%	-2.3755%
14	-2.3323%	-2.4959%	-2.4071%	-2.5000%
15	-2.6210%	-2.4570%	-2.7292%	-2.5696%
16	-2.5111%	-2.2322%	-2.7662%	-2.2305%
17	-2.0388%	-2.1054%	-2.1141%	-2.1570%
18	-1.6480%	-1.7187%	-2.6890%	-2.5163%
19	-0.1687%	-0.1625%	-0.5135%	-0.3792%
20	-0.1687%	-0.0181%	-2.3527%	-0.7592%
21	-0.0071%	-0.0080%	-0.5272%	-0.1134%
22	-0.0038%	-0.0037%	0.0445%	0.2148%
23	-0.0010%	-0.0010%	-5.2218%	0.0058%
total	**-1.9124%**	**-3.7316%**	**2.6085%**	**3.0275%**
total without class 1	**1.9282%**	**-1.8186%**	**-3.5040%**	**-2.1424%**
financial effects	**-6.8567%**	**-14.5591%**	**14.6799%**	**13.0822%**
financial effects without class 1	**-7.5351%**	**-7.4848%**	**-7.7054%**	**-7.6267%**

Source: German Statistical Office, Wiesbaden; own calculations.

We realize that the tax revenue calculated for the unmodified tax base (tables 1) by microsimulation does not differ as much as from the one determined by group simulation as do the tax revenues assuming a modified tax base[13] (table 2). Therefore, the financial consequences of the tax base modification invoke substantially greater relative deviations between the microsimulation and the group simulation. The differences occuring in the lower income classes particularly preponderate. The relative deviations between the microsimulation and the group simulation for the overall financial effects including all income classes are -6.9% (basic tax scale) and -14.6% (splitting tax scale) using TI_i^g and are 14.7% (basic tax scale) and 13.1% (splitting tax scale) referring to TI_i . If we neglect the lowest income class, relative deviations of about -7.5 % (basic and splitting tax scale) arise in the context of the interpolation of TI_i^g . -7.7 % (basic tax scale) and -7.6% (splitting tax scale) can be found by employing TI_i . Obviously, a group simulation excluding the inaccurate values of the first class leads in principle to an underestimation of the financial effects. This finding meets the expectations since by relying on the average amount of tax base deductions per corresponding taxpayer we determine the lower boundary of the possible tax revenue shortfall.

Furthermore, table 2 clarifies that the results of the group simulation that are based on the class sum of the taxable income of the taxpayers with a negative income (TI_i^g) involve - as expected – in tendency fewer deviations from the microsimulation results than is the case in a simulation that refers to the class sum of the taxable income of all taxpayers of the class (TI_i). From this, we cannot conclude that the structure of deviation identified here will generally be observable because the (unknown) distribution of the taxpayers within a class in principle may differ by class and by the examined tax facts. This is clarified comparing the class specific results in table 2.

5.3 Tax scale simulation based on total amount of income

Most of the tables provided by the German Statistical Office on income tax, in particular the documentation of several specific tax rules, is not arranged according to size of the taxable income but rather to size classes of the "total amount of income". Then, again analyzing taxpayers that are either subject to the basic or splitting tax scales, the database can be described formally as follows:

Given is a categorization per total amount of income for j=1 to m classes with class borders [c_j, d_j], where $c_1=-\infty$, $d_1=-1$, $c_2=0$ and $d_m=\infty$.[14]

For each class j we know:

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[13] The modified taxable income is given by the taxable income increased e.g. by losses that have not yet been offset against profits.

[14] For classes j>1 the taxpayers have a taxable income greater than zero DM. The first class (j=1) contains the so called cases of loss which occur if the taxpayer has an assessed negative income. A negative value can result when determining of the sum of the earnings form different sources of income or, later in the assessment pattern, when

- the frequency h_j in class j (number of taxpayers in the class for whom a taxable income has been assessed),
- the frequency g_j of a tax fact (number of taxpayers who meet this fact) and the value G_j of this tax fact (in thousands of DM or €),
- the sum of the taxable incomes of all taxpayers in this class TI_j and
- the sum of the assessed income tax of all taxpayers in this class T_j.

Applying the group simulation model to data from tables that are arranged according to total amount of income (*TAI*) the following problem arises.[15] The distribution of taxpayers with a specific taxable income ($h_{(TI)}$) is difficult to estimate due to the fact that for the taxpayers of a *TAI*-class j only the average taxable income of the class $\overline{TI_j} = \frac{TI_j}{h_j}$ can be determined directly. The interval range $[a_i, b_i]$ of the possible taxable income of these taxpayers cannot be deduced from the *TAI* tables.

By mapping a taxpayer to a certain *TAI* class we can only determine the upper limit of the taxable income b_i as the theoretical maximum taxable income of the class by reducing the upper limit of the *TAI* class d_j by the minimum fiscal reductions, e.g. allowances for special expenses. In contrast, a theoretical lower limit for the taxable income a_i cannot be determined because the taxable income can adopt any value below the upper bound of the *TAI* class d_j due to various discounts on the total amount of income, e.g. special expenses, loss offset or extraordinary expenditures. Consequently, in this case the lower interval limit a_i (smallest possible taxable income) must be estimated roughly implying relatively high inaccuracy of the results of simulation. In order to reduce the deviations in group simulation caused by this deficit of information cross tables were provided by the German Statistical Office for our analysis. These cross tables allow us to restructure part of the aggregated data of the income tax statistics that are grouped according to total amount of income (*TAI)* and rearrange them according to classes of taxable income (*TI)*.

In these cross tables the absolute frequency of the taxpayers h_i with a taxable income in class i and the sum of the taxable income TI_i are brought together with the absolute frequency of the taxpayers h_j with a total amount of income in class j and the sum of the taxable incomes TI_j of these taxpayers. As a result, we obtain a matrix of the absolute frequencies of the taxpayers h_{ij} and the necessary sums of the taxable income TI_{ij}.

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determining the taxable income, e.g. due to the deduction of extra expenditures and extraordinary expenses (cf. German Statistical Office, 1995:10).

15 Concerning problems of defining total taxable income cf. e.g. O'Donoghue and Sutherland (1999) and Goolsbee (2000).

Using this matrix it is possible to estimate the distribution of the taxpayers with a specific taxable income from the aggregated data of the income tax statistics grouped according to the class attribute “total amount of income“, as in eqns. (1.4) and (1.5):

(1.8) $$h_{(TI)} = \frac{h_{ij}}{(b_i - a_i + 1)} + \frac{TI - a_i - \left(\frac{b_i - a_i}{2}\right)}{\left(\frac{b_i - a_i}{2}\right)} u_{ij},$$

where

(1.9) $$u_{ij} = \frac{TI_{ij} - \frac{a_i + b_i}{2} h_{ij}}{\sum_{TI=a_i}^{b_i} \frac{TI - a_i \quad \frac{b_i - a_i}{2}}{\frac{b_i - a_i}{2}}} TI\,.$$

Then, employing the discrete group simulation model the income tax revenues can be determined by equation (1.3).

A comparison of the results of the group simulation based on the *TAI* tables with those of the microsimulation shows that applying cross tables (prepared by the German Statistical Office) to tax scale simulation with the discrete group model provides fairly accurate results. The relative deviations between the group simulation and microsimulation results are shown in table 3. The simulation was conducted assuming the 1990 tax scale. For those taxpayers who are taxed with the basic rate a minor overestimation of the income tax with a deviation of 0.027% can be observed running the group simulation. In case of the splitting tax scale the results of the group simulation differ -0.048%, i.e. are slightly lower than for microsimulation.

The minor overestimation for the basic tax rate is caused by the underlying data of the tax base. In table 3 a comparison of the data based on the respective simulation of the tax base precedes the comparison of the aggregated income tax.

This comparison clarifies to what extent the data applied for the microsimulation that are extrapolated from the sample differ from the data in the income tax statistics applied to the group simulation. Obviously, the differences between the data determined from the sample by the German Statistical Office and the values of the basic population in the income tax statistics are very small and therefore, the simulation results are hardly affected by the structural differences in data sets. In the group simulation based on the *TAI* tables the fixed and variable reductions and discounts from the tax base are considered in line with the proceeding in sections 5.1 and 5.2. The tax revenue that would result without taking account of specific tax facts (*g, G*) can be estimated analogously to eqn.

(1.7). Here, we usually assume $h^{g}_{(TI)} = h_{(TI)} \frac{g_j}{h_j}$ because the taxable income (TI^{g}_{j}) of the taxpayers who are subject to such tax base deductions is not known. This implies that for all examined tax facts an identical distribution of the taxpayers with a specific taxable income is assumed for each *TAI* class. This simplifying procedure may lead to greater deviations when we are using data that is classified according to total amount of income (*TAI)* as database for the group simulation in comparison to data from tables that are arranged towards the taxable income (*TI)*. This is true since the assumption of an identical distribution of the taxpayers with a specific taxable income for an interval of the taxable income $[a_i, b_i]$ leads to smaller differences than for an interval of the total amount of income $[c_j, d_j]$.

Table 3: Tax scale based micro and group simulation of tax revenue using a sample of the original micro data and grouped data of *TAI* tables (1995 income distribution)

TI class no.	relative difference between					
	sample and *TAI* tables				microsimulation and group simulation results	
	number of taxpayers		taxable income		calculated income tax	
	basic tax scale	splitting tax scale	basic tax scale	splitting tax scale	basic tax scale	splitting tax scale
1	1.4916%	-0.0093%	0.0934%	0.0690%	0.0000%	0.0000%
2	0.8065%	-0.0003%	0.4750%	0.0010%	0.0000%	0.0000%
3	0.0927%	-0.0020%	0.0891%	-0.0023%	0.0702%	-1.0534%
4	0.0692%	-0.0005%	0.0687%	-0.0009%	0.0723%	-0.2947%
5	-0.0458%	0.0226%	-0.0451%	0.0233%	-0.0455%	-0.1903%
6	0.0984%	-0.0266%	0.0993%	-0.0269%	0.1007%	-0.2267%
7	0.0482%	0.0040%	0.0493%	0.0036%	0.0502%	-0.1414%
8	0.0784%	-0.0015%	0.0780%	-0.0016%	0.0777%	-0.0970%
9	0.0649%	0.0030%	0.0659%	0.0030%	0.0657%	-0.0680%
10	0.0386%	-0.0010%	0.0391%	-0.0008%	0.0447%	-0.0544%
11	0.0324%	0.0001%	0.0325%	-0.0002%	0.0338%	-0.0445%
12	0.0332%	-0.0019%	0.0333%	-0.0023%	0.0329%	-0.0392%
13	0.0073%	0.0176%	0.0073%	0.0171%	0.0070%	-0.0172%
14	0.0788%	0.0000%	0.0790%	0.0005%	0.0792%	-0.0323%
15	0.0212%	0.0000%	0.0216%	0.0001%	0.0217%	-0.0311%
16	-0.0373%	0.0077%	-0.0384%	0.0084%	-0.0383%	-0.0203%
17	0.0761%	-0.0106%	0.0754%	-0.0105%	0.0751%	-0.0378%
18	0.0235%	0.0019%	0.0228%	0.0021%	-0.0006%	-0.0384%
19	0.0064%	-0.0073%	0.0057%	-0.0075%	-0.0004%	-0.0302%
20	-0.0046%	-0.0094%	-0.0130%	-0.0072%	-0.0162%	-0.0180%
21	0.0467%	0.0000%	0.0369%	0.0000%	0.0357%	-0.0048%
22	0.0000%	0.0000%	0.0000%	0.0000%	0.0000%	-0.0021%
23	0.0000%	0.0000%	0.0000%	0.0000%	0.0001%	-0.0003%
total	**0.2782%**	**-0.0007%**	**0.0388%**	**-0.0020%**	**0.0269%**	**-0.0477%**

Source: German Statistical Office, Wiesbaden; own calculations.

5.4 Tax base deductions simulation based on total amount of income

The results in table 3 are in line with the corresponding findings for effects of vertical loss offset restrictions on tax revenues in table 4 based on data from *TAI* tables. In case of a group simulation based on the *TAI* tables the tax bases before the loss simulation already differ from those of the microsimulation because of the different underlying datasets.

Table 4: Tax base based micro and group simulation of tax revenue and the financial effects using a sample of the original micro data and grouped data of *TAI* tables in the case of vertical loss offset restrictions (1995 income distribution)

TI class No.	relative deviation between the microsimulation results of the German Statistical Office and those of the discrete group model							
	negative income		taxable income		modified taxable income		calculated income tax on modified taxable income	
	basic tax scale	splitting tax scale	basic tax scale	splitting tax scale	basic tax scale	splitting tax scale	basic tax scale	splitting tax scale
1	-0.95%	-1.34%	-75.05%	-63.24%	73.73%	147.02%	189.31%	136.41%
2	-50.78%	-37.57%	-31.18%	-18.79%	-46.11%	-31.99%	-68.99%	-59.58%
3	-13.07%	-11.43%	0.06%	-1.08%	-7.96%	-6.68%	-29.48%	-26.05%
4	9.25%	-1.37%	13.27%	1.43%	11.13%	0.25%	-5.83%	-12.02%
5	17.25%	14.01%	19.80%	8.50%	18.53%	10.45%	6.81%	2.77%
6	29.06%	13.51%	18.99%	8.29%	23.66%	10.09%	11.29%	3.09%
7	8.07%	9.75%	6.03%	8.85%	6.91%	9.20%	-3.36%	2.21%
8	-7.58%	0.06%	-4.93%	2.19%	-5.88%	1.68%	-12.15%	-3.06%
9	-11.46%	5.50%	-3.57%	0.41%	-5.95%	1.39%	-11.73%	-1.15%
10	9.25%	16.26%	8.10%	6.99%	8.37%	8.56%	4.61%	6.37%
11	1.33%	58.41%	10.69%	23.48%	8.74%	29.04%	4.37%	28.36%
12	1.42%	40.82%	0.07%	14.03%	0.31%	18.20%	-1.98%	17.20%
13	22.78%	20.50%	16.16%	4.24%	17.33%	6.74%	14.63%	5.40%
14	5.67%	12.35%	11.61%	0.79%	10.48%	2.56%	7.42%	0.83%
15	8.27%	-5.75%	4.16%	-6.05%	4.87%	-6.00%	2.51%	-8.32%
16	-0.56%	-23.20%	1.82%	-16.78%	1.38%	-17.81%	-1.34%	-20.12%
17	28.67%	-30.56%	13.76%	-22.04%	16.32%	-23.39%	15.32%	-25.64%
18	21.9%	-55.51%	5.5%	-33.95%	8.49%	-37.79%	9.1%	-40.88%
19	36.01%	-32.40%	16.81%	-26.38%	20.69%	-27.50%	22.11%	-27.81%
20	-24.27%	1.15%	-17.82%	-0.42%	-19.11%	-0.15%	-19.82%	-0.25%
21	75.79%	9.33%	3.77%	0.89%	16.45%	2.18%	17.94%	2.33%
22	-2.36%	47.36%	-1.76%	0.51%	-1.85%	7.03%	-1.85%	7.42%
23	-68.43%	-40.02%	-15.47%	1.35%	-20.20%	-2.53%	-20.31%	-2.59%
total	**-0.17%**	**-0.35%**	**19.30%**	**5.31%**	**26.87%**	**9.74%**	**2.80%**	**-1.21%**
total without class 1	**0.55%**	**0.18%**	**0.25%**	**-1.20%**	**21.85%**	**5.37%**	**-3.15%**	**-3.95%**
financial effects							**16.47%**	**2.31%**
financial effects without class 1							**-5.09%**	**-8.79%**

Source: German Statistical Office, Wiesbaden; own calculations.

This is caused by the fact that the taxable income of the taxpayers with a negative income (TI_j^g) cannot be derived from the *TAI* tables. The tax base of the taxpayers who obtained a negative income must be estimated using the taxable income of all taxpayers (TI_j) of the respective class. The number of taxpayers with a negative income (g_j) and the sum of the negative income (G_j) per class are given in the *TAI* tables. Therefore, we realize only slight differences for the total sum of the negative incomes with -0.17% applying the basic tax rate and -0.35% for the splitting tax scale.

Concentrating on the single classes it turns out that the transition of *TAI* classes for the negative income to *TI* tables can invoke severe deviations in the individual classes. This is due to the fact that running a group simulation based on *TAI* tables the distribution of the average amount of the negative income in a *TAI* class (G_j) is made according to the distribution of the taxable income of all taxpayers of this class. Consequently, the taxable income and the modified taxable income in each single class may also show strong deviations. Here, however, the unmodified taxable income neglecting the class of the taxpayers with a taxable income less than one DM can be determined relatively exactly. We find a deviation of 0.25% applying the basic tax scale and of -1.2% applying the splitting tax scale.

Thereby, we acknowledge the findings of group simulation on basis of *TI* tables simulating tax revenue effects of reduced loss offset allowances (see table 2) for data from *TAI* tables (table 4). For the class of taxpayers with a taxable income of less than one DM only very inaccurate results can be obtained. Consequently, this leads again to an overestimation when determining the total financial effects of reduced loss offset using the group model. If we exclude the class of the taxpayers with negative income from the analysis we receive an underestimation of the financial effects of 5.1% in the case of the basic tax scale and 8.8% in the case of the splitting tax scale. These deviations are similar to the differences realized by applying *TI* tables (table 2).

6 Summary

We compare the results obtained by the microsimulation with those from a discrete group model on basis of differently classified data. Thereby, we point out and quantify the possible effects of the simplified procedure of the group model as well as the loss of information using aggregated and incomplete data. The differences identified by concentrating on specific examples do not provide generally validated values. Nevertheless, they indicate the magnitude of possible inaccuracies caused by a group simulation.

Summarizing, we find that applying the group simulation model to analyze tax scale effects leads to very good results. The differences between the results received by microsimulation in comparison to group simulation increase if we determine the financial effects of modified tax bases, particularly if tax base cuts vary between taxpayers and if we take account of the class of the taxpayers with a taxable income of less than one DM. Neglecting this class we identify a systematic underestimation simulating the financial consequences of a modified tax base with the group model assuming a progressive tax scale. Then, as disaggregated data on the tax base modification is not available we have to adjust the empirical class average of the tax base reduction. If the group simulation data is not arranged according to the taxable income but rather to the total amount of income we tend to find greater deviations from the microsimulation results in sum as well as per class.

From this we can conclude that a microsimulation should always be employed if possible. But we have to consider that the calculations of the German Statistic Office may lead to very large deviations as they rely on a sample and not on complete micro data sets. Deviations arise from the structure of the stratified random sample (on the specifics of the structure of the sample from the income tax statistics cf. Zwick, 1998b:570-571). This is not always representative of circumstances and facts with a relatively small frequency. Then, the possible errors due to the group model and the aggregated database are considerably smaller than in case of a microsimulation because in the group simulation we apply data that is based on the overall population in the income tax statistics.

Aiming to determine and analyze the tax revenue effects of alternative tax settings, in particular the financial effects of specific fiscal regulations, the group simulation model introduced here represents a good compromise between a) allow the model and the data to reflect a complex situation as accurately as possible and b) the possible accuracy of a model that is based on limited resources and data. It is beneficial to simulate the tax revenue effects of relatively simple factors applying the discrete group simulation model. Furthermore, with respect to specific questions for which the sample is not representative and thus not appropriate, for reasons of cost and reliability of the results, group simulation often is preferable to a sample-based microsimulation. Especially for analyses of rather old data our group simulation model will be of major interest as sufficiently detailed data for micro analyses is usually missing.

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