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# COSTS OF CAPITAL AND WEALTH TAX – REMARKS ON BJERKSUND AND SCHJELDERUP (2022)

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**Abstract** This note incorporates wealth taxes in a simple asset valuation model based on discounted cash flows. Any valuation method requires an adjustment of pre-tax into post-tax costs of capital. By adopting the adjustment procedure proposed in previous literature, we show that arbitrage opportunities can occur – which is incompatible with a consistent valuation. In particular, such problems arise if a wealth tax system applies that uses current (instead of previous) stock prices as the basis for assessment. Furthermore, in this setting we derive a consistent relation between pre-tax and post-tax costs of capital that is compatible with the premise of no-arbitrage by exploiting risk-neutral probabilities.

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# 1 Introduction

Bjerksund and Schjelderup (2022) recently addressed the issue of how income and wealth taxes affect the value of a share in this journal. Under a set of simplifying assumptions they came to the conclusion that these taxes have no influence and can thus be disregarded in the valuation of shares. In particular, they argue that these taxes reduce the cash flow from the asset, but at the same time this reduction is perfectly compensated by a lower discount rate, i.e., cost of capital.

The topic of the impact of taxes on stock prices is by no means new and has been discussed in the literature many times.[1] While the majority of studies focuses on income taxes, only few consider wealth taxes. It could be argued that wealth taxes are less important as most OECD countries have either abolished them or levy them exclusively on buildings and land. But this should not be a convincing argument for ignoring wealth taxes – it could be that they will once again become a more attractive instrument of tax policy in the future.[2]

The paper of Bjerksund and Schjelderup (2022) is an exception in this respect as the authors incorporate both, income and wealth taxes into their valuation calculus. However, regarding wealth taxes the discussion can by no means be taken as closed. We are convinced that the results of Bjerksund and Schjelderup (2022) hold true only for a special system of wealth taxes, which exhibits properties that are not observed necessarily in the real world. Therefore, the results must be reevaluated when taking into account alternative wealth tax systems. This is the intention of our note.

In particular, we formulate the corresponding tax base differently so that it reflects actual conditions in existing tax regimes. We then use the cost of capital relation suggested by Bjerksund and Schjelderup (2022)[3]

1. A review of the impact of dividend and capital gain taxes on asset prices is provided in Hanlon and Heitzman (2010, chapter 5). Regarding wealth taxes Advani and Tarrant (2021) review empirical evidence on behavioral responses to the incentives that arise from taxing wealth.
2. Currently, several US states are considering introducing a wealth tax, see "A national wealth tax has gone nowhere. Now some states want to tax the ultra-rich", CBS News as of January 23rd 2023. For an overview of existing and proposed wealth tax systems, see e.g., Scheuer and Slemrod (2021). Experience with taxing wealth in Europe is investigated in detail in Saez and Zucman (2022).
3. See equation (1) in Bjerksund and Schjelderup (2022, p. 876). Note, that for our purposes we abandoned the income tax and use a slightly different notation. A similar relation can also be found in Maiterth and Sureth-Sloane (2021, p. 14) however not in the context of discounted cash flow valuation. Instead they derive a minimum pre-tax return under income and wealth taxation in order to compensate inflation effects.

$$k^{\tau} = k - \tau \tag{1}$$

in order to determine the value of the company's stock. This relation assumes a simple linear relation between the cost of capital before wealth taxes $k$ and after wealth taxes $k^{\tau}$.[4] In what follows, we will provide evidence that this ad hoc assumption is not compatible with the no arbitrage principle in our setting. And it must be clear that whenever arbitrage opportunities arise any valuation model becomes inherently inconsistent.

The outline of our study is as follows: In the next section we will introduce the notation and specify our assumptions, in particular regarding the wealth tax which we include in the valuation model. We will also highlight tax properties modeled differently in comparison to Bjerksund and Schjelderup (2022) in order to reflect other existing wealth tax regimes. Section 3 develops a strategy that generates arbitrage profits resulting from applying equation (1). Finally, section 4 derives consistent after-tax costs of capital that leave no room for arbitrage opportunities and thus, allow for a consistent valuation in our wealth tax setting. The last section summarizes the results.

# 2 Notation and Assumptions

## 2.1 General Assumptions

We assume a discrete time model with time points $t = 0, 1, 2 \ldots$. The information set available at $t$ is denoted by $\mathcal{F}_t$.

Capital can be invested in two types of assets traded on the market:

1. A *riskless bond* generates a risk-free return amounting to $r_f$ that is paid to the investor periodically. As is common in finance literature the value of the bond $B_t$ is thus assumed to be constant over time[5], $B_t = B_{t+1}$.

2. A *risky asset* which generates an uncertain dividend in $t$ amounting to $CF_t$. For the sake of simplicity and in line with Bjerksund and Schjelderup (2022) we assume

4. In contrast, Fama (2021, p. 8) assumes "expected returns are the same before and after imposition of wealth taxes" which results in falling asset prices.
5. Alternatively, we could model an accumulating bond that does not pay out but reinvest the interest payments, $B_{t+1} = (1 + r_f)B_t$. Results remain qualitatively unchanged and are available on request.

that these cash flows follow a martingale process

$$\mathrm{E}[CF_{t+1}|\mathcal{F}_t] = CF_t. \tag{2}$$

with an infinite time horizon. The price of the risky asset at time $t$ is denoted by $V_t$. The costs of capital $k$ in absence of taxes are defined as

$$k := \frac{\mathrm{E}[CF_{t+1} + V_{t+1}|\mathcal{F}_t]}{V_t} - 1 \tag{3}$$

and are assumed to be constant over time. From equations (2) and (3) we obtain the simple valuation formula

$$V_t = \frac{E[CF_{t+1} + V_{t+1}|\mathcal{F}_t]}{1+k} = \lim_{T\to\infty} \sum_{s=t+1}^{T} \frac{\mathrm{E}[CF_s|\mathcal{F}_t]}{(1+k)^{s-t}} = \frac{CF_t}{k}. \tag{4}$$

If taxes are taken into account, expected cash flows, costs of capital and thus market value might change, and we therefore introduce the symbols $k^\tau$ and $V_t^\tau$, respectively. In this case, the definition (3) must by adequately extended by taxes. Bjerksund and Schjelderup (2022) suggest the relation $k^\tau = k - \tau$ to relate the post-tax cost of capital to the pre-tax cost of capital. In the following we will challenge this relation and will provide proof that it allows for arbitrage opportunities in our tax setting.

Furthermore, the capital market is efficient and no trading costs apply. A risk-neutral measure $Q$ exists which allows valuation based on the fundamental theorem.[6] Without consideration of taxes this reads as follow,

$$V_t = \frac{\mathrm{E}_Q[CF_{t+1} + V_{t+1}|\mathcal{F}_t]}{1 + r_f}.$$

## 2.2 Tax Assumptions

In order to facilitate our considerations, we will consider a wealth tax only and will completely disregard income tax. We assume the following wealth tax properties:

6. A first version of this theorem goes back to Harrison and Kreps (1979). For details see, e.g., Kruschwitz and Löffler (2020, p. 37).

*Tax subject:* Wealth tax is paid by the (domestic) investor. No exemptions and no foreign investors are taken into account.

*Tax objective:* Both types of capital market investments (risky and riskless assets) are subject to taxation.

*Tax rate:* We assume a linear tax function with wealth tax rate $\tau$ being constant over time.

*Tax base:* Taxation is assumed to be based on market values, rather than on (amortized) book values or typified non-market values. However, this does not specify the wealth tax base sufficiently: The tax base at a specific point in time $t$ can be either the current market values $V_t$ for the stock and $B_t$ for the bond. Alternatively, the prices from the previous period $V_{t-1}, B_{t-1}$ could be taken as tax bases.

Notably, the former alternative (where current market values serve as tax base) is often used in real world tax legislations: For example, the wealth tax on securities in Spain is due on December, 31st of each year, using the average market value of the fourth quarter as the basis for assessment.[7] Also, the tax legislation of the Swiss canton of Bern requires an assessment at the closing price in December for listed shares, which corresponds to the end of the tax period and thus current market values as tax base.[8]

In contrast to that, the alternative where previous market values serve as tax base is also applied in practice:[9] To determine wealth tax due in Norway, listed shares are valued on January, 1st of the year of assessment.[10] Also in Liechtenstein, the market value of securities at the beginning of the tax year is crucial for determining the taxable assets.[11]

If both determinations of the tax base are given in practice it seems a matter of taste which one to use in a model. Bjerksund and Schjelderup (2022) and Fama

7. See Art. 15 and Art. 29 of Law 19/1991 of June, 6th 1991 on wealth tax.
8. See Art. 49 para. 1, Art. 67 para. 2 and Art. 72 para. 1 of the Tax Act (StG).
9. For non-listed shares, the Swiss canton of Bern levied wealth taxes on the shares' "intrinsic value", see Art. 49 para. 2 StG. Only until 2021, this was determined on the basis of the previous year's financial statements, which is equivalent to a valuation based on previous values and thus covers the other case.
10. The assessment is made at 80% of the market value on January, 1st see § 4-12 of Act on tax on wealth and income (Tax Act).
11. See Art. 12 para. 1c) of Act of September 23, 2010 on National and Local Taxes (Tax Act; SteG).

(2021) have opted for the previous value. It can be shown that no arbitrage opportunities can occur in this constellation if one relies on the cost of capital relation (1).[12] But if one shifts to the case where the tax base is given by the current value[13] and if one further sticks to the relation (1), then – and this will be the main result in our next section – arbitrage opportunities cannot be ruled out. But a price system that allows arbitrage opportunities is inconsistent and thus worthless.

Therefore, the correct modeling of the wealth tax is crucial. We believe that the main culprit in this situation is not given by the definition of the tax base (which is modeled after legal tax provisions) but rather by using equation (1) which must be used with extreme care.

## 3 Arbitrage Strategy

Wealth tax in $t+1$ is levied on the market values in time $t+1$. Consequently, the cash flows in $t+1$ from an investment in the risky asset is reduced by the tax payment $\tau V_{t+1}^{\tau}$. Analogous to equation (4), we now obtain

$$V_t^{\tau} = \frac{E[CF_{t+1} + V_{t+1}^{\tau} - \tau V_{t+1}^{\tau} | \mathcal{F}_t]}{1+k^{\tau}} \implies V_t^{\tau} = \lim_{T\to\infty} \sum_{s=t+1}^{T} \frac{E[CF_s|\mathcal{F}_t]\,(1-\tau)^{s-t-1}}{(1+k^{\tau})^{s-t}} = \frac{CF_t}{k^{\tau}+\tau} \quad (5)$$

for the market value of the risky asset under consideration of a wealth tax.[14]

Comparing (4) to (5) reveals that the market values with and without taxes indeed equal,

$$V_t^{\tau} = V_t \quad (6)$$

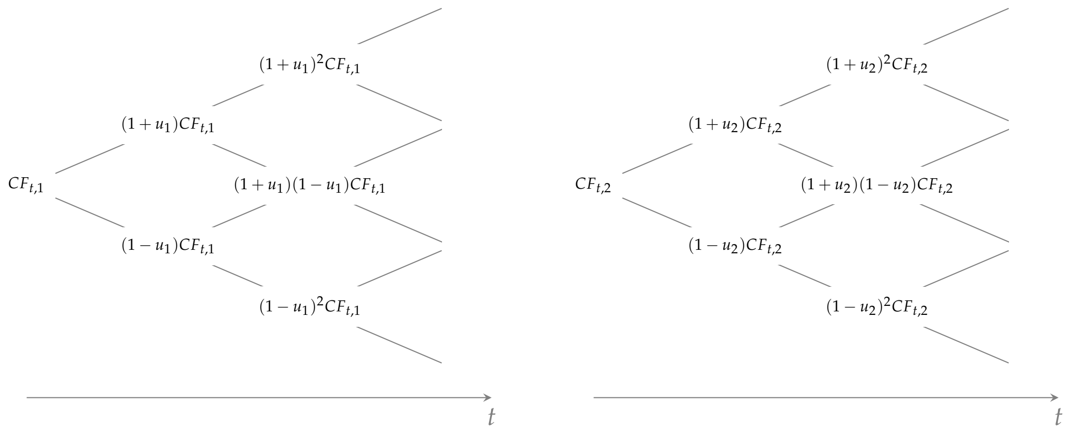
if the cost of capital relation $k^{\tau} = k - \tau$ holds true as proposed by Bjerksund and Schjelderup (2022). However, we will now show that this relation is problematic in our setting as it introduces arbitrage opportunities that destroy consistent pricing.

12. The proof is available on request.
13. See, e.g., Stowe (2020, p. 15).
14. This follows from

$$V_t^{\tau} = \lim_{T\to\infty} \sum_{s=t+1}^{T} \frac{E[CF_s|\mathcal{F}_t]\,(1-\tau)^{s-t-1}}{(1+k^{\tau})^{s-t}} = \frac{CF_t}{(1-\tau)} \lim_{T\to\infty} \sum_{s=t+1}^{T} \frac{(1-\tau)^{s-t}}{(1+k^{\tau})^{s-t}}.$$

In order to do so, we extend our model in the following way: While there is only one riskless bond in the market, we allow for several risky assets with the properties outlined in section 2. In particular, there are at least two risky assets (named 1 and 2). Moreover, we assume that pre-tax cash flows of the assets follow a binomial process with constant growth factors $u_1$ and $u_2$ according to figure 1. Up- and down-movements occur with the same probability.

*Figure 1: Cash flows of the risky assets*




The values and payments under consideration of wealth tax of both assets are shown in table 1. Note, that market values $V^\tau_{t+1}$, $B_{t+1}$ serve as tax bases at time $t+1$.

*Table 1: Assets in the Market*

| | Value in $t$ | Payments and Value in $t+1$ |
|---|---|---|
| riskless bond | $B_t$ | $(1+r_f)B_t - \tau B_{t+1}$ |
| risky asset 1 | $V^\tau_{t,1}$ | $CF_{t+1,1} + V^\tau_{t+1,1} - \tau V^\tau_{t+1,1}$ |
| risky asset 2 | $V^\tau_{t,2}$ | $CF_{t+1,2} + V^\tau_{t+1,2} - \tau V^\tau_{t+1,2}$ |

We now have a look at a portfolio consisting of $n_S$ shares in asset 2 and $n_B$ riskless bonds such that it perfectly duplicates the cash flows of risky asset 1 in $t+1$.[15] To this end we will solve the equation

$$n_S \cdot (CF_{t+1,2} + V^\tau_{t+1,2} - \tau V^\tau_{t+1,2}) + n_B \cdot ((1+r_f)B_t - \tau B_{t+1}) \stackrel{!}{=} CF_{t+1,1} + V^\tau_{t+1,1} - \tau V^\tau_{t+1,1}$$

15. This duplication procedure goes back to Kruschwitz and Löffler (2004) who identify arbitrage opportunities in a setting with income taxes.

Since (6) holds, this simplifies to

$$n_S \cdot (1+k_2-\tau)V_{t+1,2} + n_B \cdot ((1+r_f)B_t - \tau B_{t+1}) \stackrel{!}{=} (1+k_1-\tau)V_{t+1,1}$$

with the solution

$$n_S = \frac{V_{t,1}(1+k_1-\tau)u_1}{V_{t,2}(1+k_2-\tau)u_2} \tag{7}$$

$$n_B = \frac{V_{t,1}(1+k_1-\tau)(u_2-u_1)}{\left((1+r_f)B_t - \tau B_{t+1}\right)u_2}. \tag{8}$$

Now consider a portfolio consisting of the duplication portfolio long and the risky asset 1 short. At $t+1$ this portfolio has a cash flow of zero by construction. At $t$, it yields a cash flow of

$$\Delta := n_S \cdot V_{t,2} + n_B \cdot B_t - V_{t,1}$$

To eliminate arbitrage opportunities, $\Delta$ must be zero. Insertion of (8) and (7) yields

$$\Delta = \underbrace{V_{t,1}\,(1+k_1-\tau)}_{\text{term 1}} \underbrace{\left(\frac{1}{1+r_f-\tau} - \frac{1}{1+k_1-\tau} + \frac{u_1}{u_2}\left(\frac{1}{1+k_2-\tau} - \frac{1}{1+r_f-\tau}\right)\right)}_{\text{term 2}} \tag{9}$$

This equation can only take a value of zero if either term 1 or term 2 vanishes; and this is exactly the case if

$$\tau = 1+k_1 \quad \text{or} \quad \tau = 1+k_1 + \frac{(k_1-k_2)(k_1-r_f)}{\frac{u_1}{u_2}(k_2-r_f)+r_f-k_1} \tag{10}$$

holds. What does that mean? There are two and only two tax rates for which arbitrage opportunities are impossible. At all tax rates that deviate from condition (10), arbitrageurs can become as rich as they want.

## 4 Consistent cost of capital relation

In section 3 we have seen that we cannot simply assume the cost of capital relation $k^\tau = k - \tau$. It depends on the characteristics of the underlying tax system whether this relationship is appropriate in the sense that there are no arbitrage opportunities or there are. At least as long as the tax base in $t$ comprises the market values of the assets at the same point in time $t$, the cost of capital relation (1) creates arbitrage opportunities and is thus without use.[16]

The purpose of this section, therefore, is to develop a consistent relation between $k^\tau$ and $k$ that ensures no arbitrage opportunities in our wealth tax setting. We will do so, by making use of risk-free probability measure $Q$ which is assumed to by unaffected by taxation. The valuation equation (5) can be stated both in terms of the real probability measure or, alternatively, in terms of the risk-neutral probability measure.

$$\lim_{T\to\infty}\sum_{s=t+1}^{T}\frac{E[CF_s|\mathcal{F}_t](1-\tau)^{s-t-1}}{(1+k^\tau)^{s-t}} = V_t^\tau = \lim_{T\to\infty}\sum_{s=t+1}^{T}\frac{E_Q[CF_s|\mathcal{F}_t](1-\tau)^{s-t-1}}{(1+r_f-\tau)^{s-t}} \tag{11}$$

Since the cash flows form a martingale, a special version of auto-regression, it is analogously also possible to determine the present value of a separate cash flow by means of either the real probability measure or the risk-neutral probability measure, alternatively.[17]

$$\frac{E_Q[CF_s|\mathcal{F}_t]}{(1+r_F)^{s-t}} = \frac{E[CF_s|\mathcal{F}_t]}{(1+k)^{s-t}} \implies E_Q[CF_s|\mathcal{F}_t] = E[CF_s|\mathcal{F}_t]\frac{(1+r_f)^{s-t}}{(1+k)^{s-t}} \tag{12}$$

Insertion of (12) into the rhs of (11) finally results in

$$V_t^\tau = \lim_{T\to\infty}\sum_{s=t+1}^{T}\frac{E[CF_s|\mathcal{F}_t](1-\tau)^{s-t-1}}{\left((1+k)\left(\frac{1+r_f-\tau}{1+r_f}\right)\right)^{s-t}}. \tag{13}$$

Therefore, equating the lhs of (11) and (13) delivers the correct cost of capital relation

$$\begin{aligned} 1+k^\tau &= (1+k)\left(\frac{1+r_f-\tau}{1+r_f}\right) \\ \implies \qquad k^\tau &= k-\tau-\frac{k-r_f}{1+r_f}\,\tau \end{aligned}$$

and is obviously different from (1). It can easily be seen that under the commonly used assumption $r_f \in (0,k)$

$$k-\tau-k\,\tau < k^\tau < k-\tau$$

16. In contrast, a wealth tax system where the market values in $t$ are subject to taxation one period later in $t+1$ provides consistent results. Proof available on request.
17. See Kruschwitz and Löffler (2020, p. 56).

holds true. Consequently, the after-tax costs of capital are smaller than proposed by Bjerksund and Schjelderup (2022), but never fall below $k - \tau - k\,\tau$.

## 5 Summary

The aim of this study was to incorporate a wealth tax system in the valuation of risky assets where the assessment of wealth better reflects existing tax legislations. In particular, we modeled a wealth tax base that covers the current market values rather than market values from the previous point in time.

Every valuation under consideration of taxes requires the knowledge of the after-tax cost of capital $k^\tau$ in order to discount future expected after-tax cash flows. Prior literature suggested to use the relation $k^\tau = k - \tau$ that simply deducts the wealth tax rate $\tau$ from the pre-tax cost of capital $k$. However, such an ad-hoc relation is not unproblematic: We show that it creates arbitrage opportunities in our setting and is thus unsuitable for discounting cash flow. Instead, we derive a consistent cost of capital relation that ensures the no arbitrage principle.

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